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Dr. J. W. GLAISHER, M.A., F.R.S., President, in the Chair.

John Walter Lawrence Child, Vernham, Merton Hall Road, Wimbledon ;
José Comas Solá, S. Felipe, 29 S. Gervasio, Barcelona, Spain ;
Rev. John Horsley Haslam, M.A., Vicar of St. Saviour's, Denmark Park, S.E. ;
Joseph Lunt, B.Sc., F.I.C., Royal Observatory, Cape of Good Hope ; and
Joges Chandra Ray, M.A., Professor of Physical Science, Cuttack, Bengal, India,

were balloted for and duly elected Fellows of the Society.

The following Candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Alexander Eugen Conrady, F.R.M.S., 104 Park Street, Camden Town, N.W. (proposed by Edmund J. Spitta) ;
Henry Tresawna Gerrans, M.A., Fellow and Tutor of Worcester College, Oxford (proposed by Arthur R. Rambaut) ; and
Maurice Edmond Joseph Gheury, 23 Downshire Hill, Hampstead, N.W. (proposed by Silvanus P. Thompson).

One hundred and ten presents were announced as having been received since the last meeting, including, amongst others :—

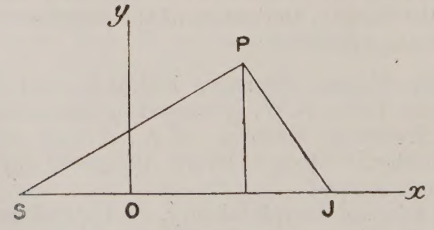
A. Auwers, Mittlere Örter von 570 Sternen aus den auf der Sternwarte Greenwich unter Direction von Pond in den Jahren 1811–1819 angestellten Beobachtungen abgeleitet (presented by the author) ; Professional papers of the Survey of India Department, No. 5, containing Major S. G. Burrard's paper on the attraction of the Himalaya Mountains (presented by the Survey Department).

On Perodic Orbits in the Restricted Problem of Three Bodies.

By E. T. Whittaker, M.A.

1. *Introduction.*—In a paper published in the *Monthly Notices*, vol. lxii. pp. 186–193, two properties of periodic orbits were communicated. The periodic orbits discussed in that paper were the closed curves which can be described by a body moving under the influence of any number of centres of force. It was supposed that these centres of force are fixed, so that the energy of the body can be partitioned into kinetic energy, which is a homogeneous quadratic function of the components of velocity, and potential energy, which does not involve the components of velocity or the time at all.

The application of periodic orbits, which is most important from the astronomical point of view, is the application to the restricted problem of three bodies (the orbits of Hill, Poincaré, and Darwin). This problem consists in the determination of the motion of a small planet P under the attraction of the Sun S and a large planet J, it being assumed that the motion of S and J is purely circular, and that the whole motion takes place in one plane. In this problem the conditions are not so simple as in the class of problems studied in the previous paper, since S and J are not fixed but *moving* centres of force, and the theorems here stated cannot be applied.



The object of the present note is to show that the methods of the previous paper can nevertheless be extended to the restricted problem of three bodies, and that two results can thus be

obtained which are analogous to, though differing from, those given in the previous paper.

2. *Formation of the equation of Least Action.*—Of the attracting bodies let S be of mass m_1, and J of mass m_2. Let O be their centre of gravity, which we can regard as fixed. Let x and y be the co-ordinates of P, when the point O is taken as origin, with the moving line OJ as axis of x, and a line perpendicular to it as axis of y. Let x be the angular velocity of OJ, and let

$$\mathrm{F}(x, y) = \frac{m_1}{\mathrm{SP}} + \frac{m_2}{\mathrm{JP}}.$$

Then the differential equations of motion of the problem are

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial \mathrm{L}}{\partial \dot{x}}\right) - \dfrac{\partial \mathrm{L}}{\partial x} = 0 \\ \dfrac{d}{dt}\left(\dfrac{\partial \mathrm{L}}{\partial \dot{y}}\right) - \dfrac{\partial \mathrm{L}}{\partial y} = 0, \end{cases}$$

where the Lagrangian function L is given by the equation

$$\mathrm{L} = \tfrac{1}{2}\{(\dot{x} - ny)^2 + (\dot{y} + nx)^2\} + \mathrm{F}(x, y),$$

and $\dot{x}$, $\dot{y}$ denote $\dfrac{dx}{dt}$, $\dfrac{dy}{dt}$ respectively.

The corresponding integral of energy is

$$\tfrac{1}{2}(\dot{x}^2 + \dot{y}^2) - \frac{n^2}{2}(x^2 + y^2) - \mathrm{F}(x, y) = h,$$

where h is the constant of energy.

Now write,

$$y' = \frac{dy}{dx} = \frac{\dot{y}}{\dot{x}},$$

and write

$$\Omega = \tfrac{1}{2}\{(\dot{x} - ny)^2 + (y'\dot{x} + nx)^2\} + \mathrm{F}(x, y),$$

so that it is equal to L, but is expressed differently,

Now the integral of energy can be written

$$\dot{x}\frac{\partial \Omega}{\partial \dot{x}} - \Omega = h,$$

and this equation can be regarded as giving $\dot{x}$ in terms of x, y, y', and h. Differentiating it on this supposition, we have

$$\dot{x}\frac{\partial^2 \Omega}{\partial \dot{x}^2}\frac{\partial \dot{x}}{\partial y'} + \dot{x}\frac{\partial^2 \Omega}{\partial \dot{x}\partial y'} - \frac{\partial \Omega}{\partial y'} = 0$$

and

$$\dot{x}\frac{\partial^2 \Omega}{\partial \dot{x}^2}\frac{\partial \dot{x}}{\partial y} + \dot{x}\frac{\partial^2 \Omega}{\partial \dot{x}\partial y} - \frac{\partial \Omega}{\partial y} = 0.$$

Now let $L' = \frac{\partial \Omega}{\partial \dot{x}}$, where L′ is supposed expressed in terms of x, y, and y', so that

$$L' = n(xy' - y) + (1 + y'^2)^{\frac{1}{2}} \{n^2(x^2 + y^2) + 2F(x, y) + 2h\}^{\frac{1}{2}}.$$

Then the two last equations become

$$\dot{x}\frac{\partial L'}{\partial y'} - \frac{\partial \Omega}{\partial y'} = 0$$

and

$$\dot{x}\frac{\partial L'}{\partial y} - \frac{\partial \Omega}{\partial y} = 0,$$

or

$$\frac{\partial L'}{\partial y'} = \frac{\partial L}{\partial \dot{y}}$$

and

$$\frac{\partial L'}{\partial y} = \frac{1}{\dot{x}}\frac{\partial L}{\partial y}.$$

The equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \frac{\partial L}{\partial y} = 0$$

becomes therefore

$$\frac{d}{dx}\left(\frac{\partial L'}{\partial y'}\right) - \frac{\partial L'}{\partial y} = 0,$$

and this single equation, which connects y and x and does not involve t, is the differential equation of those orbits for which the constant of energy is h.

To this equation there corresponds a statement of the principle of least action, namely that the integral

$$\int L' dx$$

has a value which is stationary for the actual trajectory between two given terminal points, as compared with neighbouring curves joining the same two terminal points.

3. *A criterion for the discovery of periodic orbits.*—Now consider any simple closed curve in the plane of xy; and let another simple closed curve C′ be drawn, enclosing C and differing only slightly from it. We may regard C′ as defined by an equation of the form

$$\delta p = \text{a function of } \phi,$$

where δp is the normal distance between the curves C and C′, and ϕ is the inclination of this normal distance to the axis of x.

Then if I be the value of the integral

$$\int L' dx$$

when the integration is taken round the curve C, and if $I+\delta I$ denote the value of the same integral when the integration is taken round the curve C′ (so that the symbol δ denotes an increment obtained in passing from C to C′), we find, by a method similar to that used in the previous paper, that

$$\delta I = \int \delta p \, . \, K \, . \, \{(dx)^2+(dy)^2\}^{\frac{1}{2}},$$

where, writing

$$\{h+F(x, y)+\tfrac{1}{2}n^2(x^2+y^2)\}^{\frac{1}{2}} = G(x, y),$$

we have

$$K = \frac{G(x,y)}{\varphi} + \cos\phi \frac{\partial G(x,y)}{\partial x} + \sin\phi \frac{\partial G(x,y)}{\partial y} + \frac{n}{\sqrt{2}}.$$

This equation shows that if the quantity K is positive at all points of *C*, then δI is positive, and, so the quantity I has its value increased when any curve surrounding *C* and adjacent to *C* is taken instead of C as the path of integration.

Now, suppose that another simple closed curve D can be drawn enclosing *C*, and such that at all points of D the quantity K is negative. Then in the same way it can be shown that the quantity I is increased when any simple closed curve D′, enclosed by D and adjacent to D, is taken instead of D as the path of integration.

It follows, therefore, by reasoning similar to that used in the previous paper, that, among the aggregate of simple closed curves situated in the ring-shaped space bounded by C and D, there exists a curve L, for which the value of the integral I is greater than for all the other curves of the aggregate. The curve L thus furnishes a stationary value of the integral of least action, and so is an orbit in the restricted problem of three bodies. We thus obtain theorem

If one closed curve be enclosed by another closed curve, and if the quantity K *be positive at all points of the inner curve and negative at all points of the outer curve, then in the ring-shaped space between the two curves there exists a periodic orbit of the restricted problem of three bodies, having the given arbitrary constant h as its constant of energy.*

As the quantity K can be immediately calculated for every point on any given curve, this result furnishes a means of detecting the presence of periodic orbits. It may be regarded as analogous to those methods of finding the position of roots of an algebraic equation, which depend on considerations relating to the sign of some expressions connected with the equation.

4. *An integral connected with the periodic orbits.*—Consider now the integral

$$\frac{1}{2\pi}\iint\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h+F(x, y)+\tfrac{1}{2}n^2(x^2+y^2)\}\right] dxdy,$$

where the integration is taken over the interior of any periodic orbit C in the restricted problem of three bodies. As the integrand is infinite at the points S and J we shall first suppose that, if S and J are in the interior of the orbit, small circles γ and γ' are drawn around them. If S and J are not enclosed by the periodic orbit the circles γ and γ' will be non-existent; but in order to secure the greatest generality we shall assume that they exist.

Then, if S denote the region which is contained within C but is exterior to γ and γ', we have (by Green's theorem on the conversion of integrals taken over an area into integrals taken round the boundary of the area) the equality

$$\frac{1}{2\pi}\iint_{S}\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h+F(x,y)+\tfrac{1}{2}n^2(x^2+y^2)\}\right]dxdy$$

$$=\frac{1}{2\pi}\left[\int_{C}-\int_{\gamma}-\int_{\gamma'}\right]\begin{vmatrix} dy\,\dfrac{\partial}{\partial x}\log\{h+F(x,y)+\tfrac{1}{2}n^2(x^2+y^2)\} \\ -dx\,\dfrac{\partial}{\partial y}\log\{h+F(x,y)+\tfrac{1}{2}n^2(x^2+y^2)\}\end{vmatrix}$$

the integrals round γ and γ' being inserted only when γ and γ' exist.

But the integral previously denoted by δI must be zero when the integration is taken along an arc of an actual orbit; and this (since the quantity δp which occurs in δI is arbitrary) shows that the equation

$$K=0$$

or $$\frac{1}{\rho}+\cos\phi\frac{\partial\log G(x,y)}{\partial x}+\sin\phi\frac{\partial\log G(x,y)}{\partial y}+\frac{n}{\sqrt{2}G(x,y)}=0$$

is satisfied at every point of the orbit; that is, the equation

$$2\{dx^2+dy^2\}^{-1}d\phi+\left(\cos\phi\frac{\partial}{\partial x}+\sin\phi\frac{\partial}{\partial y}\right)\log\{h+F(x,y)$$

$$+\tfrac{1}{2}n^2(x^2+y^2)\}+\sqrt{2}n\{h+F(x,y)+\tfrac{1}{2}n^2(x^2+y^2)\}^{\frac{1}{2}}=0$$

or

$$2d\phi+dy\frac{\partial}{\partial x}\log\{h+F(x,y)+\tfrac{1}{2}n^2(x^2+y^2)\}$$

$$-dx\cdot\frac{\partial}{\partial y}\log\{h+F(x,y)+\tfrac{1}{2}n^2(x^2+y^2)\}+2n\,dt=0$$

is satisfied at every point of the curve C. We have therefore

$$\frac{1}{2\pi}\int_C \left(dy\frac{\partial}{\partial x}-dx\frac{\partial}{\partial y}\right)\log\{h+F(x,y)+\tfrac{1}{2}n^2(x^2+y^2)\}$$

$$=-\frac{1}{2\pi}\int_C (2d\phi+2ndt)$$

$$=-2-\frac{\Gamma}{\pi},$$

where Γ is the angle through which the whole system of S, J, and P has turned in space during one period of the periodic orbit.

We have next to consider the integrals which arise from the curves γ and γ'. We shall take first the integral arising from the curve γ. We shall suppose that the centre of γ is at S, and that its radius is a small quantity f.

Then in the neighbourhood of S we have

$$h+F(x,y)+\frac{n^2}{2}(x^2+y^2)=\frac{m_1}{SP}+\text{terms finite in the neighbourhood;}$$

thus
$$\left(dy\frac{\partial}{\partial x}-dx\frac{\partial}{\partial y}\right)\log\left\{h+F(x,y)+\frac{n^2}{2}(x^2+y^2)\right\}$$
$$=-dy\frac{OS+x}{SP^2}+dx\,.\,\frac{y}{SP^2}+\text{less important terms.}$$

Therefore

$$-\frac{1}{2\pi}\int_\gamma \left(dy\frac{\partial}{\partial x}-dx\frac{\partial}{\partial y}\right)\log\left\{h+F(x,y)+\frac{n^2}{2}(x^2+y^2)\right\}$$
$$=\frac{1}{2\pi\rho^2}\int_\gamma\{(OS+x)dy-ydx\}+\text{infinitesimal terms}$$
$$=\frac{1}{2\pi\rho^2}\times\text{twice the area of }\gamma$$
$$=1.$$

Similarly the integral arising from γ' has the value unity.

This result is clearly independent of the radius of the small circles γ and γ', provided these are infinitesimal; we are thus entitled to speak of the integral of the quantity

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\left\{h+F(x,y)+\frac{n^2}{2}(x^2+y^2)\right\}$$

taken over the interior of the periodic orbit, since the arbitrary

element of choice in the circles γ and γ' does not affect the value of the integral. We thus obtain the result—

The integral

$$\frac{1}{2\pi}\iint\left[\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\log\{h+\mathrm{F}(x,y)+\tfrac{1}{2}n^2(x^2+y^2)\}\,dxdy,\right.$$

where the integration is taken over the interior of any periodic orbit in the restricted problem of three bodies, has the value

$$-\frac{\Gamma}{\pi},\ \text{or}\ -1-\frac{\Gamma}{\pi},\ \text{or}\ -2-\frac{\Gamma}{\pi},$$

according as the periodic orbit encloses both, one, or none of the bodies S *and* J*; where* Γ *is the angle through which the whole system composed of* S, J, *and* P *turns in space during one period of the periodic orbit.*

1902 *January* 24.

On the Images formed by a Parabolic Mirror. First Paper: The Geometrical Theory. By H. C. Plummer, M.A.

1. *Introduction.*—As is well known the staple work of the University Observatory at Oxford during the past few years has been the execution of its share of the Astrographic Chart. At the same time lines of investigation suggested by this work have not been neglected, and much has been done in the discussion of the systematic errors to which photographs are liable. As an example of such researches reference may be made to Professor Turner's papers on the optical distortion of the photographic doublet (*Monthly Notices*, vol. lix. p. 438).

Soon after I became associated as assistant with the observatory Professor Turner suggested that, as I had already paid some attention to the forms of the images produced by a reflecting telescope (*A. J.*, No. 435) I should extend and as far as possible complete this study. Accordingly I undertook the direct comparison of a plate kindly lent by Dr. Isaac Roberts with one of our own astrographic plates. This task is proceeding and will, I hope, shortly be finished. In the meantime a comparison of my own earlier paper with various other theoretical papers revealed apparent discrepancies which it is desirable to remove by a thorough revision of this part of the subject. The results of this investigation are independent of the actual comparison of the two plates, and it seems convenient therefore to publish them separately.

2. My earlier paper, "On the Star Image formed by a Parabolic Mirror," was published in the *Astronomical Journal*,

No. 435, simultaneously with one by Mr. J. M. Schaeberle. Previously, in 1887, the subject had been discussed by General Tennant (*Monthly Notices*, vol. xlvii. p. 244). A fourth paper, by Mr. C. W. Crockett (*Astrophysical Journal*, vol. vii. p. 362), will also receive notice. Curiously enough the first two and the last of these papers all bear the date 1898 March. Differences of notation and variety of method have made it necessary to collate these papers with considerable care. The outcome of the examination may be briefly indicated here. In General Tennant's paper an unfortunate slip is pointed out which seems to have escaped detection hitherto, and which makes his final results only an approximation to the truth. I have found that Mr. Schaeberle's formulæ are right, though left in a rather inconvenient form, if we correct an apparent misprint. On the other hand his figures of the image formed by different zones of the mirror are certainly incorrect in detail. In Mr. Crockett's results, so far as they go, there is nothing calling for criticism; on the contrary, I have found his curves, evidently drawn with great care, most interesting. In my own paper rigorous expressions were found for the equations of the reflected ray. The further development of the theory was based on a clearly defined approximation which led in a remarkably simple manner to results sufficiently in accord with the actual appearance of star images. The degree of approximation adopted is easily examined, but no examination was made before and the omission will be rectified in the present paper. The analysis will also be extended so as to consider the form of the image in a plane close to, but not coincident with, the focal plane, a step which has not hitherto been taken in any paper, so far as I am aware.

3. *General Theory.*—I retain as far as possible the notation of my previous paper. Let the equation of the surface of the mirror be

$$4fx = y^2+z^2 = r^2.$$

Let the plane ZOX be the principal plane, *i.e.* the plane containing the axis of the mirror and the ray incident at O, the vertex, and let θ be the angular distance of the star from the centre of the field, being in the positive direction from OX. The direction cosines of the normal at P, the point of incidence (x, y, z), are

$$\frac{2f}{\sqrt{4f^2+r^2}}, \quad \frac{-y}{\sqrt{4f^2+r^2}}, \quad \frac{-z}{\sqrt{4f^2+r^2}}.$$

We may then put

$$r = 2f\tan\tfrac{1}{2}v, \; y = r\sin\phi, \; z = r\cos\phi.$$

If F is the focus, v is the angle OFP and ϕ is the angle

between the planes POX and ZOX. The direction cosines of the normal become

$$+\cos\tfrac{1}{2}v, \quad -\sin\tfrac{1}{2}v\sin\phi, \quad -\sin\tfrac{1}{2}v\cos\phi.$$

If χ is the angle between the incident ray and the normal we have

$$\cos\chi = \cos\tfrac{1}{2}v\cos\theta - \sin\tfrac{1}{2}v\cos\phi\sin\theta$$

Let (λ, μ, ν) be the direction cosines of the reflected ray. Now the direction cosines of the incident and reflected rays may also be regarded as the coordinates of points at unit distance from O along lines parallel to those rays. And the direction cosines of the normal multiplied by $\cos\chi$ are the coordinates of a point at a distance $\cos\chi$ from O along a line parallel to the normal. But this point is clearly midway between the other two, and by expressing this fact we obtain

$$\begin{aligned}
\lambda + \cos\theta &= 2\cos\tfrac{1}{2}v\,[\cos\tfrac{1}{2}v\cos\theta - \sin\tfrac{1}{2}v\cos\phi\sin\theta]\\
\mu &= -2\sin\tfrac{1}{2}v\sin\phi\,[\cos\tfrac{1}{2}v\cos\theta - \sin\tfrac{1}{2}v\cos\phi\sin\theta]\\
\nu + \sin\theta &= -2\sin\tfrac{1}{2}v\cos\phi\,[\cos\tfrac{1}{2}v\cos\theta - \sin\tfrac{1}{2}v\cos\phi\sin\theta].
\end{aligned}$$

Hence

$$\left.\begin{aligned}
\lambda &= \cos v\cos\theta - \sin v\sin\theta\cos\phi\\
\mu &= -\sin v\cos\theta\sin\phi + \sin^2\tfrac{1}{2}v\sin\theta\sin 2\phi\\
\nu &= -\sin v\cos\theta\cos\phi + \sin^2\tfrac{1}{2}v\sin\theta\cos 2\phi - \cos^2\tfrac{1}{2}v\sin\theta
\end{aligned}\right\} \quad (1)$$

4. Now let the reflected ray, whose direction cosines have just been found, meet the focal plane in the point (ξ, η, ζ). Then,

$$\xi = f, \qquad \eta = y + \frac{\mu}{\lambda}(f-x), \qquad \zeta = z + \frac{\nu}{\lambda}(f-x).$$

Remembering that

$$f - x = f(1-\tan^2\tfrac{1}{2}v) = f\cos v\sec^2\tfrac{1}{2}v\,;$$

we then find

$$\begin{aligned}
\lambda\eta &= 2f\tan\tfrac{1}{2}v\sin\phi\,(\cos v\cos\theta - \sin v\sin\theta\cos\phi)\\
&\quad + f\cos v\sec^2\tfrac{1}{2}v\,(-\sin v\cos\theta\sin\phi + \sin^2\tfrac{1}{2}v\sin\theta\sin 2\phi)\\
&= -f\tan^2\tfrac{1}{2}v\sin\theta\sin 2\phi.\\
\lambda\zeta &= 2f\tan\tfrac{1}{2}v\cos\phi\,(\cos v\cos\theta - \sin v\sin\theta\cos\phi)\\
&\quad + f\cos v\sec^2\tfrac{1}{2}v\,(-\sin v\cos\theta\cos\phi + \sin^2\tfrac{1}{2}v\sin\theta\cos 2\phi\\
&\qquad - \cos^2\tfrac{1}{2}v\sin\theta) = -f\tan^2\tfrac{1}{2}v\sin\theta\cos 2\phi - f\sin\theta.
\end{aligned}$$

The co-ordinates may therefore be written in the form

$$\left.\begin{aligned}
\eta &= -\frac{f\tan\theta}{\cos v}\cdot\frac{\tan^2\tfrac{1}{2}v\sin 2\phi}{1-\tan v\tan\theta\cos\phi}\\
\zeta &= -\frac{f\tan\theta}{\cos v}\cdot\frac{1+\tan^2\tfrac{1}{2}v\cos 2\phi}{1-\tan v\tan\theta\cos\phi}
\end{aligned}\right\} \quad \ldots \quad \ldots \quad (2)$$

These simple expressions make a very convenient basis for a study of the aberration in the focal plane of a parabolic mirror.

5. Let us now put

$$d = -f \tan\theta \sec v, \qquad a = \tan^2 \tfrac{1}{2}v, \qquad b = \tan v \tan\theta.$$

Then the equations found for the co-ordinates may be written

$$\left.\begin{aligned} \frac{\eta}{d} &= \frac{a \sin 2\phi}{1 - b\cos\phi} \\ \frac{\zeta}{d} &= \frac{1 + a\cos 2\phi}{1 - b\cos\varphi} \end{aligned}\right\} \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

and these may be regarded as equations in terms of a third variable ϕ of the geometrical image formed in the focal plane by a circular zone of the mirror. They are in the form most convenient for the discussion of the properties of the curve which they represent, and make it easy to plot the curve for any assigned values of the constants. It is merely of mathematical interest to eliminate ϕ and so obtain the ordinary Cartesian equation. It is convenient first to move the origin from the centre of the field along the axis of ζ to the point $\zeta = d(1-a)$. When this is done the result of elimination is :

$$\begin{aligned} &(\eta^2+\zeta^2)[\eta^2\{(1-a)b^2+2a\}^2+4a^2(1-b^2)\zeta^2] \\ &\quad + 2d\zeta[\eta^2\{b^2(1-a)^2(b^2\overline{1-a}+2a)+2aJ\} - 4a^2\zeta^2(b^2\overline{1-a}+2a)] \\ &\quad + d^2J\{b^2(1-a)^2\eta^2 - 4a^2\zeta^2\} = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4) \end{aligned}$$

where

$$J = b^2(1-a)^2 - 4a^2.$$

The curve is therefore a quartic.

6. We can now draw from (2) some simple inferences with regard to the properties of the curves formed by the rays reflected at a circular ring of the mirror. In the first place we see, by changing the sign of ϕ, that the curve is symmetrical about a radius through the centre of the field, a fact which is obvious *a priori* and from the form of equation (4). On the trace of the principal plane or axis of symmetry we have the following scheme of values :—

$$\phi = 0, \quad \eta = 0, \quad \zeta = -f\frac{\tan\theta \,.\, \sec v \sec^2\frac{1}{2}v}{1 - \tan v \tan\theta} = d \,.\, \frac{1+a}{1-b} \ldots \quad (A)$$

$$\phi = \frac{\pi}{2}, \quad \eta = 0, \quad \zeta = -f\tan\theta \,.\, \sec^2\tfrac{1}{2}v = d(1-a) \quad \ldots \quad (B)$$

$$\phi = \pi, \quad \eta = 0, \quad \zeta = -f\frac{\tan\theta \,.\, \sec v \sec^2\frac{1}{2}v}{1 + \tan v \tan\theta} = d \,.\, \frac{1+a}{1+b} \ldots \quad (C)$$

$$\phi = \frac{3\pi}{2}, \quad \eta = 0, \quad \zeta = -f\tan\theta \,.\, \sec^2\tfrac{1}{2}v = d\,(1-a) \quad \ldots \quad (B)$$

It may be remarked that investigations confined to the principal plane are quite inadequate, inasmuch as they fail to explain the genesis of the image in the principal plane itself. The point B is clearly a double point lying on the two branches of the curve which correspond to $\phi=\frac{\pi}{2}$ and $\phi=\frac{3\pi}{2}$. This also is shown by the Cartesian equation, which gives for the tangents at the point

$$\left.\begin{aligned} &b(1-a)\eta \pm 2a\zeta=0 \\ \text{or} \quad &\tan\theta\,.\,\eta \pm \tan\tfrac{1}{2}v\,.\,\zeta=0 \end{aligned}\right\} \quad \ldots \quad \ldots \quad (5)$$

This would indicate that the tangents are equally inclined to the axes when $v=2\theta$. It will be noticed, however, that J, the numerical coefficient of the terms of the lowest degree, vanishes at the same time. It is necessary to observe therefore that the terms of the third degree possess the terms of lowest degree as a factor when the coefficient vanishes, and continuity is thus preserved. I have purposely thrown the terms of the third degree into a form which reveals this fact on inspection. Thus when $v=2\theta$ there is a triple point through which pass the branches corresponding to $\phi=\frac{\pi}{2}$, $\phi=\pi$, and $\phi=\frac{3\pi}{2}$. When $v>2\theta$, the points BC A are in order of increasing distance from the centre of the field and from the point I corresponding to the ray reflected at the vertex of the mirror.

7. It is important now to examine the maximum and minimum values of the coordinates. For a critical value of η

$$2\cos 2\phi\,(1-\tan v\tan\theta\cos\phi)=\tan v\tan\theta\sin\phi\sin 2\phi$$

$$\therefore\ 2\cos 2\phi-\tan v\tan\theta\cos 2\phi\cos\phi=\tan v\tan\theta\cos\phi$$

$$\therefore\ \cos 2\phi=\tan v\tan\theta\cos^3\phi \quad \ldots \quad \ldots \quad (6)$$

Now $\tan v\tan\theta=b$ is always small in a practical case. To take an extreme instance, one of Dr. Common's mirrors has a focal length of 3·7 feet and an aperture of 1·7 feet. For the outside ring $v=13°$, and if $\theta=2\frac{1}{2}°$ b only amounts to ·01. Hence we are justified in taking $\phi=\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ or $\frac{7\pi}{4}$ as the solution of (6), though a closer approximation to the roots can easily be made if desired. The points where η is a maximum or minimum are therefore given by

$$\phi=\frac{\pi}{4}\ \text{or}\ \frac{7\pi}{4},\ \eta=\pm d\,.\,\frac{a}{1-b/\sqrt{2}},\ \zeta=d\,.\,\frac{1}{1-b/\sqrt{2}} \quad \ldots \quad (\text{E, E}')$$

$$\phi=\frac{3\pi}{4}\ \text{or}\ \frac{5\pi}{4},\ \eta=\pm d\,.\,\frac{a}{1+b/\sqrt{2}},\ \zeta=d\,.\,\frac{1}{1+b/\sqrt{2}} \quad \ldots \quad (\text{D, D}')$$

Since b is small the width of the curve at the points where ζ has these two values is practically the same, viz. $2ad$.

Again, for a critical value of ζ,

$$-2\tan^2\tfrac{1}{2}v\sin 2\phi\,(1-\tan v\tan\theta\cos\phi)$$
$$=\tan v\tan\theta\sin\phi\,(1+\tan^2\tfrac{1}{2}v\cos 2\phi)\quad\ldots\quad\ldots\quad\ldots\quad(7)$$

This is satisfied by $\sin\phi=0$, *i.e.* $\phi=0$ or π. The curve at the points A and C crosses the axis of ζ at right angles. The factor $\sin\phi$ being omitted, the equation reduces to

$$2\tan^2\tfrac{1}{2}v\tan v\tan\theta\cos^2\phi-4\tan^2\tfrac{1}{2}v\cos\phi$$
$$-\tan v\tan\theta\,(1-\tan^2\tfrac{1}{2}v)=0$$

or
$$2ab\cos^2\phi-4a\cos\phi-b\,(1-a)=0$$
$$\therefore\ 2ab\cos\phi=2a\pm\sqrt{[4a^2+2a(1-a)b^2]}$$
$$\therefore\ 2ab\cos\phi=2a\pm[2a+\tfrac{1}{2}b^2(1-a)-\ldots]$$

The upper sign is impossible, for it would make $\cos\phi>1$. The lower sign, if higher powers of b be neglected, gives

$$\cos\phi=-b\,(1-a)/4a=-\tan\theta/2\tan\tfrac{1}{2}v.$$
$$\therefore\phi=\pi\pm\cos^{-1}\frac{\tan\theta}{2\tan\tfrac{1}{2}v}\quad\ldots\quad(\mathrm{H},\ \mathrm{H}')$$

8. These values of ϕ give minimum values of ζ which become impossible when v is small, the limit occurring when $v=\theta$ approximately. After the disappearance of these minima as the centre of the mirror is approached the true minimum is given by $\phi=\pi$. It is then theoretically important to determine v so as to make ζ an absolute minimum. Now

$$\zeta=-f\tan\theta\frac{\sec^2\tfrac{1}{2}v}{\cos v+\sin v\tan\theta}$$

Hence for a minimum

$$\sec^2\tfrac{1}{2}v\tan\tfrac{1}{2}v(\cos v+\sin v\tan\theta)=\sec^2\tfrac{1}{2}v(-\sin v+\cos v\tan\theta)$$
$$\therefore\tan\tfrac{1}{2}v(1-\tan^2\tfrac{1}{2}v+2\tan\tfrac{1}{2}v\tan\theta)+2\tan\tfrac{1}{2}v$$
$$-(1-\tan^2\tfrac{1}{2}v)\tan\theta=0$$
$$\therefore\tan\theta=\frac{3\tan\tfrac{1}{2}v-\tan^3\tfrac{1}{2}v}{1-3\tan^2\tfrac{1}{2}v}=\tan\tfrac{3}{2}v$$
$$\therefore v=\tfrac{2}{3}\theta.$$

This value of v leads to

$$\zeta=-f\sin\theta\sec^3\tfrac{1}{3}\theta.$$

Also it is easy to see that the maximum value of ζ corresponds to $\phi=0$ and is by (A)

$$\zeta=-f\sin\theta\,.\,\sec^2\tfrac{1}{2}v\sec(v+\theta),$$

which again is greatest when v corresponds to the edge of the mirror. The total length of the complete geometrical image is therefore

$$l=f\sin\theta\,[\sec^2\tfrac{1}{2}v\sec(v+\theta)-\sec^3\tfrac{1}{3}\theta]\quad\ldots\quad(8)$$

and if powers and products of v and θ of the fifth order be neglected the expression obtained becomes

$$l = \tfrac{1}{12} f\partial(3v+2\theta)^2.$$

But this expression is of purely theoretical interest and does not apply strictly to a reflector as adapted for photographic use, since the central part of the mirror is cut off by the plate-holder and thus a part of the image is lost (*see* §15).

9. *On General Tennant's Paper* (*Monthly Notices*, vol. xlvii. p. 244).—I follow here General Tennant's own notation. He writes down (p. 250) the equations of the reflected ray thus : in the plane XY_0

$$y_0 - av = \frac{4c-4v-cv^2}{4+4cv-v^2}\left(x-\frac{av^2}{4}\right)$$

and in the plane XZ_0

$$Z_0 = \frac{(av^2}{4} - x)\tan\alpha\sin\phi.$$

I say that the latter equation is incorrect. Simple geometrical considerations would show that this is so, but perhaps the most satisfactory method will be to find the right equation which ought to be substituted for the preceding.

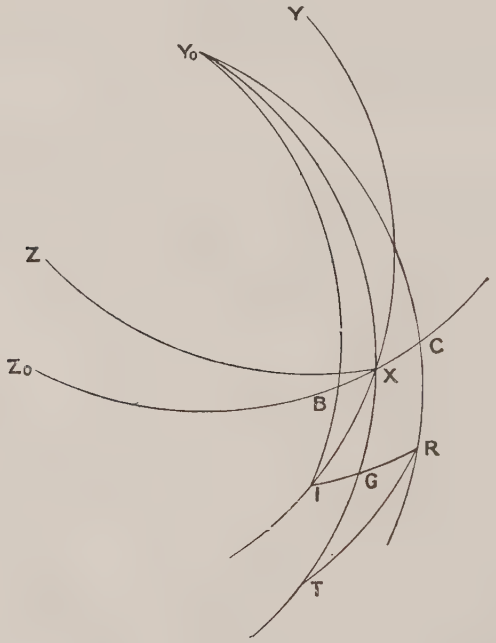


FIG. 1.

Now the point of incidence P_r of the ray on the mirror is $(\frac{1}{4}av^2, av, 0)$, the axes being $X_o\ Y_o\ Z_o$. If then $v = 2\tan\psi$, ψ is the angle between the axis of the mirror and the normal at P_r. We are chiefly concerned with the directions of certain lines, and we therefore construct a spherical figure (fig. 1) by drawing parallels through the centre of a sphere. Let X Y Z, X Y_o Z_o correspond to the two sets of axes and I, G, R to the incident ray, the normal and the reflected ray respectively. We notice that YXI, Y_oXG, and IGR lie on the same great circles and that IG = GR. Let Y_oI and Y_oR cut Z_oX in B and C, and let $BX = BY_oX = \beta$ and $XC = XY_oC = \gamma$. The points B and C correspond to the projections of the incident and reflected rays on the plane XZ_o. Hence the equations of these projections are :

$$z_o = (x - \tfrac{1}{4}av^2)\tan\beta$$
$$z_o = (\tfrac{1}{4}av^2 - x)\tan\gamma.$$

On Y_oG produced we take T so that $GT = XG = \psi$. Then the triangles XGI, TGR are altogether equal, and consequently $RT = IX = \alpha$ and $GTR = GXI = YXY_o = \phi$. Hence from the triangle Y_oTR, in which $Y_oT = \frac{\pi}{2} + 2\psi$ we get

$$-\sin 2\psi \cos\phi = \cos 2\psi \cot\alpha - \sin\phi \cot\gamma.$$

And from the triangle Y_oXI

$$0 = \cot\alpha - \sin\phi \cot\beta.$$

We have then

$$\tan\beta = \sin\phi \tan\alpha$$

and

$$\tan\gamma = \frac{\sin\phi}{\cos 2\psi \cot\alpha + \sin 2\psi \cos\phi}$$
$$= \frac{(4+v^2)\sin\phi \tan\alpha}{4 + 4cv - v^2}$$

We see then that General Tennant has in effect made the tacit assumption that $\gamma = \beta$. If we avoid this error and proceed with the transformation by means of the formulæ

$$y = y_o \cos\phi - z_o \sin\phi$$
$$z = y_o \sin\phi + z_o \cos\phi$$

we shall obtain the equations of the reflected ray in the proper form and arrive at results equivalent to those found in §§ 3 and 4. The oversight is most unfortunate, as it detracts from an otherwise admirable paper, which, so far as I am aware, contains the first serious discussion of the subject under consideration.

10. *On Mr. Schaeberle's Paper* (*A. J.* No. 435).—Mr. Schaeberle's paper is based on a very elementary geometrical method which does not, however, tend to simplify the results. His

notation is retained in this review of his theory. The principal formulæ are

$$\frac{x}{F}=\sec^3\tfrac{1}{2}v\left[\sin\tfrac{1}{2}v\cos A+\frac{\cos v\sin\tfrac{1}{2}\phi\cos(180^\circ+A+B)}{\sin(I-\tfrac{1}{2}\phi)}\right] \quad (15\,S)$$

$$\frac{y}{F}=\sec^3\tfrac{1}{2}v\left[\sin\tfrac{1}{2}v\sin A+\frac{\cos v\sin\tfrac{1}{2}\phi\sin(180^\circ+A+B)}{\sin(I-\tfrac{1}{2}\phi)}\right] \quad (16\,S)$$

to which must be added the subsidiary relations

$$\sin B=\frac{\tan\theta\sin A}{\sqrt{[\tan^2\tfrac{1}{2}v+\tan^2\theta-2\tan\tfrac{1}{2}v\tan\theta\cos A]}} \quad \ldots\ (7\,S)$$

$$\cos I=\sin\tfrac{1}{2}v\cos B \ldots \quad \ldots\ (8\,S)$$

$$\sin\tfrac{1}{2}\phi=\sin I\cos\theta\sqrt{[\tan^2\tfrac{1}{2}v+\tan^2\theta-2\tan\tfrac{1}{2}v\tan\theta\cos A]} \quad (9\,S)$$

The numbers attached to these equations correspond to those given in Mr. Schaeberle's paper. His equation (7), however, contains under the radical the term $-2\tan v\tan\theta\cos A$, an apparent misprint which is corrected above. I shall now show that these equations, unpromising as they may appear, can be made to give simple expressions for the coordinates in the focal plane equivalent to those already found. It is merely necessary to eliminate B, I, and ϕ. We have, from (7 S),

$$\cos B=[\tan\tfrac{1}{2}v-\tan\theta\cos A]/p$$

where $$p^2=\tan^2\tfrac{1}{2}v+\tan^2\theta-2\tan\tfrac{1}{2}v\tan\theta\cos A.$$

$$\therefore\ \cos(A+B)=[\tan\tfrac{1}{2}v\cos A-\tan\theta]/p$$
$$\sin(A+B)=\tan\tfrac{1}{2}v\sin A/p.$$

Also, by (8S),

$$\cos I=[\sin\tfrac{1}{2}v\tan\tfrac{1}{2}v-\sin\tfrac{1}{2}v\tan\theta\cos A]/p$$

$$\therefore\ \sin I=q/p.$$

where $q^2=\sin^2\tfrac{1}{2}v+\tan^2\vartheta\,(1-\cos^2A\sin^2\tfrac{1}{2}v)-\sin v\tan\theta\cos A.$

Then by (9S)

$$\sin\tfrac{1}{2}\phi=\cos\theta\,.\,q$$
$$\therefore\cos^2\tfrac{1}{2}\phi=\cos^2\theta\cos^2\tfrac{1}{2}v+\sin^2\theta\cos^2A\sin^2\tfrac{1}{2}v+\sin v\sin\theta\cos\theta\cos A$$
$$\therefore\ \cos\tfrac{1}{2}\phi=\cos\theta\cos\tfrac{1}{2}v+\sin\theta\sin\tfrac{1}{2}v\cos A.$$

Hence

$$\sin(I-\tfrac{1}{2}\phi)=[\cos\theta\cos v\sec\tfrac{1}{2}v+2\sin\theta\sin\tfrac{1}{2}v\cos A]q/p$$

$$\therefore\ \frac{\sin\tfrac{1}{2}\phi}{\sin(I-\tfrac{1}{2}\phi)}=\frac{\cos\tfrac{1}{2}v\,.\,p}{\cos v(1+\tan v\tan\theta\cos A)}.$$

We can now substitute in (15 S). Thus :

$$\frac{x}{F} = \sec^3 \tfrac{1}{2}v \left[\sin \tfrac{1}{2}v \cos A - \frac{\cos \tfrac{1}{2}v \,(\tan \tfrac{1}{2}v \cos A - \tan \theta)}{1 + \tan v \tan \theta \cos A} \right]$$

$$= \frac{\left[\sin \tfrac{1}{2}v \tan v \tan \theta \cos^2 A + \cos \tfrac{1}{2}v \tan \theta \right]}{1 + \tan v \tan \theta \cos A} \sec^3 \tfrac{1}{2}v$$

$$\therefore x = \frac{F \tan \theta}{\cos v} \cdot \frac{1 + \tan^2 \tfrac{1}{2}v \cos 2A}{1 + \tan v \tan \theta \cos A}.$$

And similarly, by substituting in (16 S), we get

$$\frac{y}{F} = \sec^3 \tfrac{1}{2}v \left[\sin \tfrac{1}{2}v \sin A - \frac{\sin \tfrac{1}{2}v \cos A}{1 + \tan v \tan \theta \cos A} \right]$$

$$= \sin \tfrac{1}{2}v \tan v \tan \theta \sin A \cos A \, \frac{\sec^3 \tfrac{1}{2}v}{1 + \tan v \tan \theta \cos A}$$

$$\therefore y = \frac{F \tan \theta}{\cos v} \cdot \frac{\tan^2 \tfrac{1}{2}v \sin 2A}{1 + \tan v \tan \theta \cos A}.$$

These formulæ for x and y become identical with those given for ζ and η in (2) when it is remembered that A is the angle which I have called ϕ and that θ is reckoned positive in opposite directions by Mr. Schaeberle and myself, *i.e.* that we assume the star to be on opposite sides of the axis of the mirror.

11. But though Mr. Schaeberle may claim that his formulæ are essentially correct, the same cannot be said of the figure 2 which appears on p. 19 of his paper. He has drawn two curves for $\theta = 30'$ corresponding to the zones for which $v = 5°$ and $v = 3°$. The larger curve in particular is far from accurate. With regard to the drawing of this curve the following points may be noticed : (1) it is obviously of a degree higher than the fourth, for a straight line can be drawn to cut it in six points ; (2) the two loops do not cut the axis of x at right angles, but possess singular points where they cross this axis ; (3) the points for which A is 90° and 270° do not lie on the axis of x ; (4) the most serious error lies in the disparity between the width of the loops, the true difference being of the order 1 in 1000. With the exception of (3) the same criticisms apply also to the smaller figure. It is impossible for me to trace the cause of these errors. Possibly the supposed misprint to which I have referred is really a slip. It seems more likely that the responsibility rests with the ambiguous signs which occur in connection with the irrational expressions left in the equations. Finally it may be that the arithmetic is at fault. I give drawings of the curves (fig. 2), which I hope will be found more satisfactory. I venture to deprecate the tendency to refer to the simple geometrical aberration of the parabolic mirror as "Schaeberlian." With all proper respect for Mr. Schaeberle's work I cannot help regarding this title as unnecessary and inappropriate.

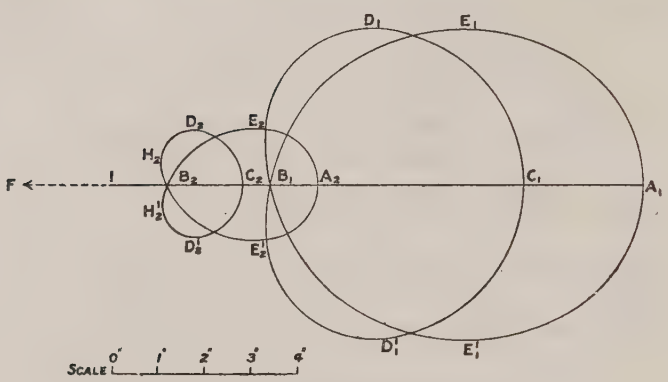


FIG. 2.

12. *On Mr. Crockett's Paper* (*Ap. J.* vii. p. 362).—I shall verify Mr. Crockett's final formulæ by deducing from them equations equivalent to (2). The set of relations marked (5) in his paper is as follows :—

$$\left.\begin{aligned} r_1 &= \frac{r \sin \alpha}{\sin (\gamma + \alpha)} \\ \cos \gamma &= \sin \theta \cos (\psi_1 - \psi) \\ \sin \gamma &= \frac{\sin (\psi_1 - \psi)}{\sin \psi} \\ \tan (\psi_1 - \psi) &= - \cos \theta \tan \psi \end{aligned}\right\} \quad \ldots \quad \ldots \quad (5\ \mathrm{C})$$

If I may permit myself a word of complaint I would say that the author's explanations err on the side of brevity. In the foregoing equations r, θ are the polar co-ordinates of the point of incidence in the plane containing this point and the axis of the mirror, the focus being the pole. Consequently

$$r = f \sec^2 \tfrac{1}{2}\theta.$$

Also r_1, ψ_1 are the polar co-ordinates of the corresponding point of the image in the focal plane, the focus of the mirror again being the pole. The inclination of the incident ray to the axis is α, and ψ is the same angle which I have previously denoted by ϕ. It is unnecessary to define the angle γ, since it will be eliminated immediately. Instead of the third relation we may write

$$\sin \gamma = - \cos \theta \sec \psi \cos (\psi_1 - \psi)$$

$$\therefore \sin (\gamma + \alpha) = - \sec \psi \cos (\psi_1 - \psi)$$

$$(\cos \theta \cos \alpha - \sin \theta \sin \alpha \cos \psi).$$

If then we put

$$R = -\frac{f \sec^2 \frac{1}{2}\theta \sin \alpha \cos \psi}{\cos \theta \cos \alpha - \sin \theta \sin \alpha \cos \psi}$$

we shall have

$$\begin{aligned} x_1 = r_1 \cos \psi_1 &= R \cos \psi_1 \sec (\psi_1 - \psi) \\ &= R \{\cos \psi - \sin \psi \tan (\psi_1 - \psi)\} \\ &= R \sec \psi (\cos^2 \psi + \cos \theta \sin^2 \psi) \\ &= R \sec \psi (\cos^2 \tfrac{1}{2}\theta + \sin^2 \tfrac{1}{2}\theta \cos 2\psi) \end{aligned}$$

and

$$\begin{aligned} y_1 = r_1 \sin \psi_1 &= R \sin \psi_1 \sec (\psi_1 - \psi) \\ &= R \{\sin \psi + \cos \psi \tan (\psi_1 - \psi)\} \\ &= 2R \sin \psi \sin^2 \tfrac{1}{2}\theta. \end{aligned}$$

Hence, replacing R by the expression which it denotes, we get immediately

$$x_1 = -\frac{f \tan \alpha}{\cos \theta} \cdot \frac{1 + \tan^2 \frac{1}{2}\theta \cos 2\psi}{1 - \tan \theta \tan \alpha \cos \psi}$$

$$y_1 = -\frac{f \tan \alpha}{\cos \theta} \cdot \frac{\tan^2 \frac{1}{2}\theta \sin 2\psi}{1 - \tan \theta \tan \alpha \cos \psi}$$

and these equations again, except for changes of notation, are identical with those numbered (2).

13. In figures 4 and 5 on p. 365 of his paper Mr. Crockett has given a number of curves obtained by assuming $\alpha = 1°$ and giving θ a series of values from 0° to 3°. Most of these curves have little or no significance in a practical study of the photographic instrument (see § 15), but their variations of form are theoretically interesting and I am glad that this part of the study can be completed by a reference to Mr. Crockett's work. I only wish to point out how his curves exemplify the theory of §§ 6–8. When $\theta = 3°$ (I follow here Mr. Crockett's notation) the resulting H curve is of the same type as those of fig. 2. When $\theta = 2° = 2\alpha$ the G curve possesses the triple point which results from the coincidence of the points B and C. As θ becomes smaller the point I have denoted by C, passing to the other side of B, approaches the true image for the central point of the mirror, and the F curve given by $\theta = 1\frac{1}{2}°$ has two loops on the side of C towards the centre of the field. The point C coincides with the true image when

$$\sec \theta \sec^2 \tfrac{1}{2}\theta = 1 + \tan \theta \tan \alpha$$

or approximately,

$$1 + \tfrac{1}{4}\theta^2 = 1 - \tfrac{1}{2}\theta^2 + \alpha\theta.$$

This gives $\theta = \frac{4}{3}\alpha$, and in fact when $\theta = 1\frac{1}{3}°$ the E curve is given passing through the point image given by $\theta = 0°$. Mean-

while the loops mentioned have disappeared and the corresponding value of θ can be estimated by identifying the maximum of y_1 at the point D with the minimum of x_1 at the point H. This gives $\frac{\tan \alpha}{2 \tan \frac{1}{2}\theta} = \cos \frac{\pi}{4}$ or $\theta = \sqrt{2} \cdot \alpha$ approximately. With $\alpha = 1°$, $\theta = 1° 24' 51''$, which agrees satisfactorily with Mr. Crockett's estimate of $1° 25'$. The complete curve for smaller values of θ consists of two loops, one of which includes the point image given by $\theta = 0°$. When $\theta = \alpha = 1°$, a case for which the D curve is drawn by Mr. Crockett, the symmetrical pair of points H, H' where x_1 is a minimum disappear, the geometrical meaning being that at the point denoted by C the curve ceases to be concave and becomes convex to the centre of the field. Finally the point of the image nearest to the centre of the field is given by $\theta = \frac{2}{3}\alpha = 40'$, and after this point is reached the curves arising from the inner zones of the mirror contract in size and approach the point which in a certain sense is the true geometrical image. Mr. Crockett's curves are therefore in agreement with what the general theory would lead us to expect.

14. *On my previous paper* (*Astronomical Journal*, *No.* 435). The coordinates of a point of the image in the focal plane are given in this form :

$$\eta = -\frac{2yz\,(f+x)\sin\theta}{(4f^2-r^2)\cos\theta - 4fz\sin\theta}$$

$$\zeta = -\frac{\sin\theta\{4f^3+2fz^2-x(y^2-z^2)\}}{(4f^2-r^2)\cos\theta - 4fz\sin\theta}$$

When x, y, and z are expressed in terms of f, v, and ϕ these become

$$\eta = -\frac{8\sin\phi\cos\phi\tan^2\frac{1}{2}v\sec^2\frac{1}{2}v}{4(1-\tan^2\frac{1}{2}v) - 8\tan\frac{1}{2}v\tan\theta\cos\phi}\cdot f\tan\theta$$

$$= -\frac{f\tan\theta}{\cos v}\cdot\frac{\tan^2\frac{1}{2}v\sin 2\phi}{1-\tan v\tan\theta\cos\phi}$$

and

$$\zeta = -\frac{\{4+8\tan^2\frac{1}{2}v\cos^2\phi+4\tan^4\frac{1}{2}v\cos 2\phi\}}{4(1-\tan^2\frac{1}{2}v) - 8\tan\frac{1}{2}v\tan\theta\cos\phi}\cdot f\tan\theta$$

$$= -\frac{f\tan\theta}{\cos v}\cdot\frac{1+\tan^2\frac{1}{2}v\cos 2\phi}{1-\tan v\tan\theta\cos\phi}.$$

These expressions are therefore, as stated, exact. In the further development of the theory the approximations employed were

$$\eta = -\frac{\tan\theta}{2f}yz$$

$$\zeta = -f\tan\theta - \frac{\tan\theta}{4f}(r^2+2z^2)$$

or, as we may now write them,

$$\eta = -f \tan\theta \,.\, \tan^2 \tfrac{1}{2}v \sin 2\phi \quad \ldots \quad \ldots \quad \ldots \quad (9)$$
$$\zeta = -f \tan\theta \{1 + \tan^2 \tfrac{1}{2}v \,(2 + \cos 2\phi)\}$$

Now the aberration contains terms of the order v^2. Hence it is in general perfectly justifiable to neglect terms of the order v^4, $v^3\theta$, or $v^2\theta^2$ for practical purposes. If this is done the approximation to η is seen to be adequate. On the other hand a proper approximation to ζ is

$$\zeta = -f \tan\theta \{1 + \tan^2 \tfrac{1}{2}v \,(2 + \cos 2\phi) + \tan v \tan\theta \cos\phi\} \quad (10)$$

which differs from the preceding expression for ζ by the addition of the term $-f \tan\theta \,.\, \tan v \tan\theta \cos\phi$.

15. The effect of this term is to make the curve of the zone image depart considerably from the two coincident circles previously found, unless θ is small compared with v. The maximum value of the term when $v = n°$ and $\theta = 1°$ is about n'', and of course it diminishes fairly rapidly towards the centre of the field. It introduces distortion apart from mere change of scale value. When $\phi = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ the term vanishes, and when ϕ is given values differing by π, the corresponding points are equidistant from the point given by (9) and the following equation. Hence the circle which is the locus corresponding to the latter equations may be regarded as a mean between the two loops represented by (9) and (10). This affords in many cases a justification for neglecting the term when v is fairly large; and when v is small, although the form of the curve as shown in the review of Mr. Crockett's paper undergoes considerable changes, the omission has little practical effect. This is due partly to the smallness of the curves and partly to the shadow which the photographic plate and its holder cast on the mirror. In the direction $\phi = \pi$ the light is evidently intercepted at least as far as the zone $v = 2\theta$. In the plane $\phi = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ the zone $v = \alpha$ is ineffective, 2α being the angular width of the field covered by the plate. The result is that the curves considered do not contribute much to the image, especially in the case of stars at some distance from the edge of the plate. The images are thus deprived of the fairly sharp point directed towards the centre of the field, and the geometrical focus for the centre of the mirror will in general lie nearer the focus of the surface than any point of the image actually formed. The precise effect in actual measurement can only be determined by experiment. With the reservation now stated I still consider the analysis of the distribution of intensity in the image, developed in my former paper, fairly satisfactory as a first approximation, and it is probably not worth while to push further a purely geometrical investigation. That examination seems certainly preferable to the merely general explanations

given elsewhere and will serve at least to emphasise the necessity of paying attention to the effect of star magnitude and time of exposure in the reduction of a photograph taken with a parabolic mirror.

16. *On the form of the image in a plane parallel to the focal plane.*—Let the reflected ray meet the plane $x = f-h$ in the point (ξ, η, ζ). Then

$$\eta = y + \frac{\mu}{\lambda}(f-h-x) = \eta_0 - \frac{\mu}{\lambda}.h$$

$$\zeta = z + \frac{\nu}{\lambda}(f-h-x) = \zeta_0 - \frac{\nu}{\lambda}.h,$$

where η_0, ζ_0 are the co-ordinates of the corresponding point in the focal plane already found in § 4. Now in the coefficients of h, which are of the first order in v, we may fairly neglect terms of the third order in v and θ. This will give as sufficient approximations to (1)

$$\mu/\lambda = -\sin\phi\tan v$$
$$\nu/\lambda = -\cos\phi\tan v - \tan\theta.$$

Then, keeping a consistent degree of approximation throughout, we use for η_0, ζ_0 the forms (9) and (10) of § 14. Thus

$$\eta = -f\tan\theta\tan^2\tfrac{1}{2}v\sin 2\phi + h\tan v\sin\phi$$
$$\zeta = -f\tan\theta\{1+\tan^2\tfrac{1}{2}v(2+\cos 2\phi) + \tan v\tan\theta\cos\phi\} + h(\tan\theta+\tan v\cos\phi).$$

Moreover, as quantities depending on h multiplied by terms of the third order in v and θ have already been neglected, it is allowable to put $f' = f-h$, and write

$$\left.\begin{aligned}\eta &= -f'\tan\theta\tan^2\tfrac{1}{2}v\sin 2\phi + h\tan v\sin\phi \\ \zeta &= -f'\tan\theta\{1+\tan^2\tfrac{1}{2}v(2+\cos 2\phi)\} - (f'\tan^2\theta - h)\tan v\cos\phi\end{aligned}\right\} \quad (11)$$

which become identical, as they should, with (9) and (10) when $h=0$, and only differ from the latter by the change of scale value indicated by f' in the place of f and by the addition of the terms $h\tan v\sin\phi$ and $h\tan v\cos\phi$ to η and ζ respectively.

17. The discussion of the locus represented by (11) may fitly be combined with that of the loci represented by (9) and the following equation and by (9) and (10), for all three loci are related in an interesting way. In the first place we put

$$\rho = 2f\tan\theta\tan^2\tfrac{1}{2}v\cos\phi \quad \ldots \quad \ldots \quad (12)$$

This is the polar equation of a circle referred to a point on its circumference, or rather of a double circle arising equally from

the variation of ϕ between 0 and π and between π and 2π. Then (9) and the following equation may be written

$$\eta = -\rho \sin \phi$$
$$\zeta = -f \tan \theta \sec^2 \tfrac{1}{2}v - \rho \cos \phi,$$

which show that the final approximations of my former paper represent a circle which, as there explained, generates a kite-shaped image of angle 60° (fig. 3*a*). Now (9) and (10) differ from the equations just considered only in the addition of the term $-f \tan^2 \theta \tan v \cos \phi$ to ζ. The nature of the modification thus introduced is easily understood, and is illustrated in fig. 3*b*. It would then be possible to deduce the character of the locus represented by (11) from the locus last considered by changing f into f' in the circular terms, $f \tan^2 \theta$ into $(f' \tan^2 \theta - h)$ in the correcting term of ζ, and finally applying to η the term $h \tan v \sin \phi$. It seems better, however, to discard the circular approximation (except as a guide to position) and to put

$$\rho = 2f' \tan \theta \tan^2 \tfrac{1}{2}v \cos \phi - h \tan v \quad \ldots \quad \ldots \quad (13)$$

This is the polar equation of a limaçon. If the term $-f' \tan^2 \theta \tan v \cos \phi$ in ζ be neglected for the moment (11) may now be written

$$\eta = -\rho \sin \phi$$
$$\zeta = -f' \tan \theta \sec^2 \tfrac{1}{2}v - \rho \cos \phi.$$

It would be tedious and unnecessary to consider possible varieties of form in detail. It is sufficient to take the case in which the limaçon is of the two-looped kind, in which case the relation to the previous double-circular approximation is obvious. The node is at the pole, which is in the same position as in the former case. The positions of two pairs of points are important. These are K, L (fig. 3*c*), given by

$$\phi = \frac{\pi}{2} \text{ and } \frac{3\pi}{2}, \quad \eta = \pm h \tan v, \quad \zeta = -f' \tan \theta \sec^2 \tfrac{1}{2}v$$

and M, N given by

$$\phi = 0 \text{ and } \pi, \quad \eta = 0, \quad \zeta = -f' \tan \theta \sec^2 \tfrac{1}{2}v - 2f' \tan \theta \tan^2 \tfrac{1}{2}v \pm h \tan v$$

These points serve to fix the position of the limaçon in relation to the circle obtained by putting $h = 0$.

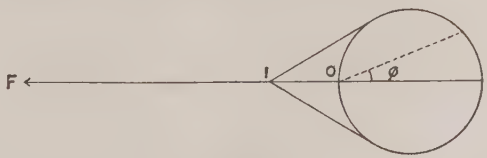


Fig. 3*a*.

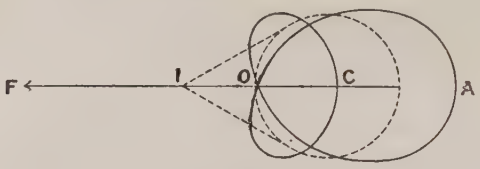


Fig. 3*b*.

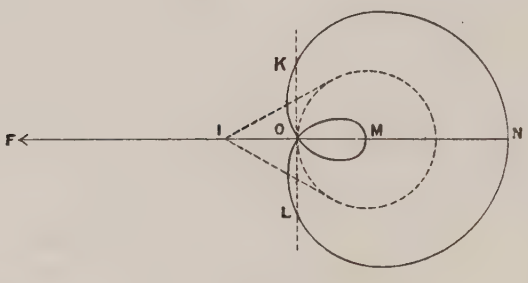


Fig. 3*c*.

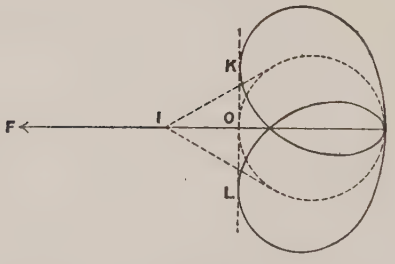


Fig. 3*d*.

18. In order to obtain the final approximation aimed at it is now necessary to replace the additional term in ζ, the general effect of which will be fairly evident. But it seems scarcely worth while to consider the result in detail. For it is clear that the addition will leave the points K, L where $\phi = \frac{\pi}{2}$ and $\frac{3\pi}{2}$ unaltered, and that it will cause the points M and N to move towards each other. The result is therefore very small, and in the most favourable case can only make the points M and N coincide at a point on the circumference of the circular approximation. In fact the point midway between M and N is unchanged in position. It does not seem then that a sufficient improvement in the outer parts of the field can be effected at the expense of the definition at the centre of the field and the lateral aberration of all the images, and it seems almost better to be satisfied with the best possible adjustment at the focus of the mirror. If, however,

special circumstances seem to make it desirable to make a compromise by adjusting the photographic plate so as to get the best definition, or rather the minimum radial aberration at a distance θ from the centre of the plate, then the proper position is given by

$$h = f \tan^2 \theta,$$

in which case the image is generated by a point whose coordinates are

$$\left.\begin{aligned} \eta &= -f' \tan\theta \tan^2 \tfrac{1}{2}v \sin 2\phi + f' \tan^2\theta \tan v \sin\phi \\ \zeta &= -f' \tan\theta \{1 + \tan^2 \tfrac{1}{2}v\,(2 + \cos 2\phi)\} \end{aligned}\right\} \quad \ldots \quad (14)$$

The use of curved plates seems also to offer no advantage sufficient to warrant the extra trouble and expense. The proper radius of curvature for such plates is now found to be

$$\mathrm{R} = f^2 \tan^2\theta / 2h = \tfrac{1}{2}f \qquad \ldots \qquad \ldots \quad (15)$$

a result which is in agreement with the well-known curvature of the field in its usual conventional sense. The impossibility of effectively overcoming the aberration of a parabolic mirror by any device of the kind considered has also been stated by Mr. S. C. Reese (*Astrophysical Journal*, vol. xii. p. 227), after an investigation of a very different kind. As the nature of the images depends on curves given by (14) a typical curve is sketched (fig. 3*d*) with the circle to which it is related in the manner already explained.

19. In conclusion a word or two of summary may be added to what has already been said in § 2 of the critical part of this paper. I still believe that my former paper (*A. J.* No. 435), which gives fig. 3*a* as a fair approximation, may be considered a satisfactory *general* explanation of the nature of the image formed by a parabolic mirror. Moreover its method and results have been of great service in the extension given to the theory in §§ 16–18 above. But confessedly accuracy has in a slight degree been sacrificed for the sake of simplicity. If then my work is to be of use in designing a parabolic mirror for a definite purpose, it will be easy to draw the necessary curves with absolute accuracy with the help of equations (2) of § 4 of the present paper. In most cases the simpler approximate forms (9) and (10) of § 14 will be sufficient. The discussion given in §§ 16–18 is necessarily brief, but it will suffice, I hope, to prove the negative result that more or less kite-shaped images are unavoidable in the case of a reflector as ordinarily adapted for photographic use.

University Observatory, Oxford:
1902 *March* 10.

The Flash Spectrum, Sumatra Eclipse, 1901 *May* 18.

By S. A. Mitchell, Ph.D.

(*Communicated by Professor J. K. Rees.*)

The transparency accompanying this paper contains :

1. Spectrum of the Second Flash.
2. Spectrum taken 5 seconds after Second Flash.

The spectra were photographed with an objective plane grating, and the positives made without enlargement.

The writer, through the courtesy of the former astronomical director of the Naval Observatory, Professor S. J. Brown, became a member of the expedition to view the Sumatra eclipse 1901 May 18.

Before reaching the island, it had been decided to occupy two stations for observations on the eclipse ; one, Solok, near the central line of totality, the other Fort de Koch, near the northern edge of the Moon's shadow path.

On our arrival in the East Indies, it was soon found that the greatest trouble was going to be clouds, for almost at no time during the day was the sky perfectly clear. It was, therefore, thought best to subdivide the party at Solok, and a third station was selected at Sawah Loento, about twenty miles distant, at the terminus of the railroad, where were already Mr. and Mrs. Newall, of Cambridge, England, and a party from the Massachusetts Institute of Technology under the direction of Professor Burton.

Two instruments were taken to Sawah Loento, a camera of 104 inches focus to be used in connection with a cœlostat for photographing the corona ; and a spectroscope consisting of a Rowland flat grating of 15,000 lines, with a ruled surface of $3\frac{1}{2}$ by 5 inches, and a quartz lens of $3\frac{23}{64}$ inch aperture and 72 inches focal length. Light from the Sun, reflected by the cœlostat mirror in a horizontal direction, fell on the grating, where it was diffracted and was brought to a focus on the photographic plate by means of the quartz lens. If grating and photographic plate are each perpendicular to the diffracted beam, the spectrum is "normal." It was arranged to photograph the first order spectrum from λ 3000 to λ 6000.

The final adjustments were made a few days before the eclipse by Mr. L. E. Jewell, the focusing being accomplished by means of a collimator designed by him.

The day of the eclipse dawned clear, and our hopes were that these favourable conditions would remain until after totality, which occurred shortly after noon. First contact was observed

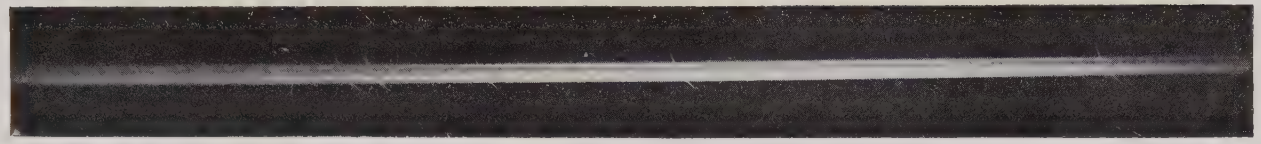
UNITED STATES NAVAL OBSERVATORY

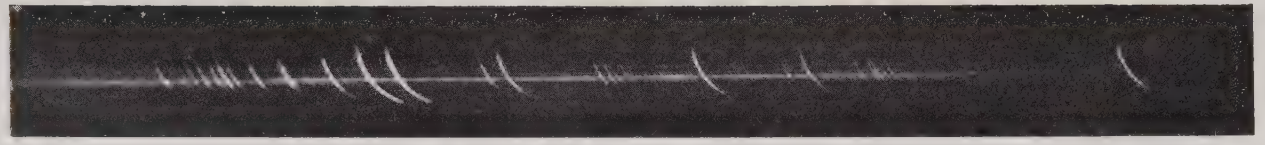
SUMATRA ECLIPSE EXPEDITION

May 18, 1901

5 sec. after Second Flash

Exposure ½ sec.





Second Flash

Exposure 3 sec.

Photographed with an OBJECTIVE PLANE GRATING

BY

S. A. MITCHELL, Columbia University Observatory, New York City

MONTHLY NOTICES, ROYAL ASTRONOMICAL SOCIETY, VOL. LXII. PLATE 8

in a perfectly cloudless sky, but soon after this clouds began to gather, and a quarter of an hour before second contact the sky was completely overcast.

The disappearing crescent of the Sun was watched by a binocular, before one half of which was arranged a small plane grating in such a way that with one eye the spectrum could be seen, with the other the Sun itself.

With this, shortly before the time of second contact, bright lines were seen for a few seconds at F and H, and in several places in the green and yellow, but these disappeared almost immediately, the Sun again being completely hidden by clouds—and the first flash passed without our being able to see it.

Towards the middle of totality conditions became a trifle better, so that it was possible to see, through clouds, the corona extending for about half a diameter from the Sun, and with the small spectroscope the "coronium line" quite distinctly. During no time of the $5^m\ 41^s$ of totality was an unclouded view of the corona obtained, but nevertheless the second flash was seen beautifully.

An hour after totality the clouds cleared away and a perfect sky remained for the rest of the afternoon!

Altogether eight exposures were made, one before and one just after totality for the cusp spectrum, one at first and one at second flash, and four with different lengths of exposure during the total phase. The first flash is entirely lacking, clouds having completely cut off all the light; the second flash seemed fully exposed, and it is to the discussion of its photograph that this paper is devoted.

The "Flash."—The peculiarities of this photograph of the flash are twofold:

1. Normal spectrum.
2. Great dispersion.

On the plate, the distance from F to H is 95·4 mm., and, as the spectrum is normal, 1 mm. therefore corresponds to a difference of wave-length of 9·37 tenth-metres, or 1 tenth-metre corresponds to a dispersion of about 0·1 mm. This is about one-fifth the dispersion obtained with the ordinary Rowland mounting with a 21½-foot grating of 20,000 lines.

For some reason the spectra were not in perfect focus, and it was thus practically impossible to measure the cusp spectra. Owing to the great dispersion, although the bright lines of the flash were not perfectly sharp, measures were made and wave-lengths determined with a high degree of accuracy.

The plate was measured by one of the Repsold machines belonging to Columbia University, by comparing the spectrum lines directly with a millimetre scale. Measures with this machine can be made directly to 0·005 mm., and by estimation to 0·0005 mm., *i.e.* to 0·005 λ. However the sharpness of the lines did not permit them to be carried to this degree of accuracy

Comparisons with the Solar Spectrum.—Those who have

attempted to identify the bright lines with Rowland's map know the difficulties of this undertaking, which arise from the differences of dispersion between the solar spectrum and the flash, but more especially from the great differences of intensities of the lines in the two spectra. Great care was exercised in the determinations of the wave-lengths and in the comparisons with Rowland's tables of standard wave-lengths.

The spectrum extends from λ 4924 to λ 3320, but the focus becomes poor at the violet end beyond K, and measures were discontinued at λ 3835, Hη. The photograph from F to H was in the best focus, and for the present purposes considerations are confined to this region.

Neglecting H and He lines, 374 lines were measured in the flash between F and H ; 91 of these were unidentified, 283 identified with lines in the solar spectrum. An arbitrary scale of intensities was assumed where 0 denotes a line seen with certainty, 10 the strongest line of the spectrum.

Two points are immediately noticed in comparing the two spectra ; first, for each and every element, the brighter the solar line the brighter the flash line corresponding to it ; second, the intensities of the solar lines which correspond to a line of given brightness in the flash differ with different metals. Fe and Ni lines of intensity 5, Ti, Sc, and V lines of intensity 2, are identified with flash lines of equal strength. These differences for the various elements were so marked that in order to arrive at their significance, and hence draw some conclusions regarding the 'reversing layer,' three tables were made. The first contains the flash lines arranged according to metals and intensities of lines (scale 0–10) ; the second the solar lines with which the flash lines were identified arranged according to metals and intensities (scale 1–1000) ; while the third was made up of all the lines in Rowland's map having an intensity greater than 2 (scale 1–1000).

Although we cannot directly compare the intensities of the bright lines of the flash (scale 0–10) with those of the dark lines given in Rowland's tables (scale 1–1000), we arrive at certain theoretical considerations if we compare the average intensities for the different elements, *i.e.* $\frac{\text{Flash intensities}}{\text{Solar intensities}}$, and also the ratios of the number of lines of each element identified in the flash to the whole number of solar lines for that metal. Forming these ratios, and arranging them, we are at once struck with the systematic variations not only in the ratios of intensities but also in the percentage of lines identified.

The meaning of these systematic differences will be understood if we consider these ratios in combination with the atomic weights of the various elements, as is done in Table I., where also are put down the number of the lines in the flash due to each metal.

TABLE I.

Group I.—Lines strong in flash, and in solar spectrum.

Element.	Atomic Weight.	Number of Lines Identified.	Intensity Flash / Intensity Solar Lines.	Number Lines Identified / Total Number Lines.
Na	23·0	—	—	—
Mg	24·3	1	0·10	1·00
Al	27·1	—	—	—
Ca	40·0	8	0·34	0·38
Group II.—Lines strong in flash, weak in solar spectrum.				
Sc	44·1	6	0·81	0·75
Ti	48·1	62	0·68	0·70
V	51·2	15	0·68	0·67
Cr	52·1	38	0·55	0·69
Mn	55·1	27	0·24	0·48
Sr	87·6	2	1 08	0·67
Y	88·7	2	0·50	0·67
Zr	90·6	9	0·46	0·70
Group III.—Lines weak in flash, strong in solar spectrum.				
Fe	56·0	125	0·23	0 32
Ni	58·7	9	0·36	0·28
Co	59·0	6	0·19	0·29

Looking at the numbers of the two last columns we see that the lines naturally fall into three groups as given in the table.

To these may also be added the following lines :

La, atomic weight 138·5, 3 lines at λ 4123·384, λ 4217·720, and λ 4613·544.

Ba, atomic weight 137, 1 line at λ 4554·211,

and the following lines possibly identified :

Si, atomic weight 12, 1 line at λ 3905·660,

Zn, atomic weight 65, 1 line at λ 4810·724,

Ce, atomic weight 92, 2 lines at λ 4003·912 and λ 4107·649.

In Group I. would also fall Al, if we consider the relative intensities of the two lines λ 3944·160 and λ 3971·674 ; and undoubtedly Na, if our plate took in the D lines.

The grouping of these lines is exactly that adopted by Evershed from his investigations of the Indian eclipse, except I have put Zr with Sr and V in Group II. Mn seems to represent the transition from Group II. to Group III.

As Evershed has pointed out, the remarkable variations of the relative intensities in the flash and Fraunhofer spectra are undoubtedly due to the heights to which the vapours of the different metals ascend in the chromosphere. We would natur-

ally expect that these heights vary according to the atomic weights of the metals, those of least atomic weights ascending to the greatest distances ; and generally speaking this no doubt is true. But if we have two gases in the Sun's atmosphere, one a gas with an intrinsic brightness 1 and 100 miles in thickness, it would give a photographic line in the flash spectrum just as bright as the other gas of intrinsic brightness 100 and only 1 mile thick, *if* the Sun and Moon were relatively at rest during the period of the flash ; but considering the gradual advance of the Moon in covering up successive layers of the Sun's atmosphere we see that in the emission spectrum the photographic brightness of the fainter gas would be many times that of the brighter. The absorption caused by a gas depends on the total number of molecules the solar ray comes in contact with, and will be very nearly equal in the two cases.

In view of these considerations it would therefore seem that the gases of the metals of Group II. extend very high, that they are nowhere very much condensed, and that practically all the gas contributes to the formation of the emission line, and hence, the flash lines are to be regarded as true reversals of the corresponding solar lines.

The vapours of Groups I. and III. are somewhat condensed near the Sun's surface (those of Group I., particularly Ca, reach far greater heights than those of Group III.), but as it is the upper portion that contributes most to the formation of the emission lines owing to the progressive motion of the Moon, the flash lines are to be regarded as only partial reversals of the Fraunhofer lines, the solar intensities being greater than the flash intensities.

Unknown Lines.—Taking account of lines in the flash identified with groups in the solar spectrum, about half the solar lines have corresponding lines in the flash. From the above considerations we see that it is highly improbable that lines of intensity 2 in the solar spectrum, and belonging to Groups I. and III., will have lines corresponding to them of sufficient brightness to show in this photograph of the flash. In fact, although there are 135 Fe lines of intensity 2 in Rowland's map between F and H, only 11 of these are found in the flash ; and indeed great numbers of the feebler solar lines have no counterpart in the flash spectrum. But if, on the other hand, we compare the stronger lines, we see that every strong line of the solar spectrum almost without exception is found in the flash.

And so, remembering the meaning of the differences of intensity, we see no reason for giving up our faith in the existence of the "reversing layer."

Columbia University, New York City :
1902 *January* 28.

The Duration of Totality at Navalmoral.

By C. T. Whitmell, M.A., B.Sc.

For the total solar eclipse of 1900 May 28 I have shown, by a comparison of a large number of observations of the predicted and observed durations of totality for several localities, that the eclipse semidiameter of the Moon, adopted by the *British Nautical Almanac*, is too large. (See *Total Solar Eclipse of* 1900, pp. 75–81.) In the present paper I propose to give some details with regard to Navalmoral, whence I observed this eclipse.

For Navalmoral the following data were published by Dr. Downing :—

Adopted position, 5° 34′ W. longitude, 39° 52′ N. latitude.

		G.M.T. h m s
Eclipse begins	May 28	2 48 45
Totality „	„	4 6 12
Totality ends	„	4 7 39
Eclipse „	„	5 15 45

	Angle from N. Point. °
First contact	273
Second „	99
Third „	267
Fourth „	93

A note was appended to the effect that the predicted duration of totality (87ˢ) is perhaps 3ˢ too long.

The duration of totality at Navalmoral was carefully timed independently by four observers, and we agreed upon a totality of 80ˢ. Mr. Howarth, F.R.A.S., the Rev. S. J. Johnson, F.R.A.S., and Mr. Southall used only the eye unaided.

I observed with a low-power binocular, of which the right object-glass was fitted with a diffraction grating attached to a prism, so that the beginning and the ending of totality were estimated by the disappearance and the reappearance of the normal solar spectrum. The coincidence in the four estimates seems to afford reasonable ground for believing that the duration of totality was, as nearly as possible, 80ˢ.

There is thus a loss of at least 7ˢ to be accounted for, and this may be due to :—(*a*) errors in the lunar tables, (*b*) error in the adopted position of Navalmoral, (*c*) error in the *N.A.* value of the Moon's geocentric semidiameter, which for eclipses is 15′ 32″·65 at mean distance. It is assumed that the *N.A.* eclipse semidiameter of the Sun is correct.

With regard to (*a*) it should be noted that, as the transit of the Moon over the Sun was practically central, it may be inferred that any allowable error in the Moon's tabular latitude would be insufficient appreciably to alter the duration of totality.

As to (*b*) Señor Iñiguez, of the Madrid Observatory, has courteously supplied the exact position of Navalmoral Church. The coordinates are 5° 32′ 29″ W. longitude, and 39° 53′ 29″ N. latitude, the difference from the originally adopted coordinates being only 1′ 31″ in longitude and 1′ 29″ in latitude.

By *N.A.* data, at mid-totality, $4^h\ 6^m\ 56^s$, G.M.T., the centre of the shadow was in 5° 31′ 45″ W. longitude, and 39° 53′ 40″ N. latitude, coordinates differing by only 44″ of longitude and 11″ of latitude from the position of the church as given by Señor Iñiguez. As we were about half a mile N. of the church we were only some 20″ of latitude N. of the central line, so small a distance that for us the transit of the Moon over the Sun was practically central during totality.

We have now to consider (*c*). Adopting for Navalmoral the data supplied by Señor Iñiguez, and using *N.A.* data for the R.A., Dec., and semidiameters of the Sun and Moon, I have made for $4^h\ 0^m$, $4^h\ 7^m$, and $4^h\ 10^m$, G.M.T., a careful computation of the positions of the Sun and Moon, and obtain a totality duration of 88^s, or just one second in excess of the *N.A.* predicted value. The moment of mid-totality agrees almost exactly with the *N.A.* time, $4^h\ 6^m{\cdot}56^s$. The nearest approach of the centres of the Sun and Moon is about 0″·3, so that, as already pointed out, the transit was practically central. The position angle for second contact is about 93°, that for third contact is about 271°, reckoned eastward from the Sun's north point.

Thus we have really to account for a loss of about 8^s. It will be found that to reduce the duration of totality to 80^s the effective apparent eclipse semidiameter of the Moon, as seen by the observer during totality, must be reduced from its *N.A.* value of 16′ 5″·71 to 16′ 3″·97, a reduction of 1″·74; whilst the geocentric mean semidiameter at the time of totality must be reduced from 15′ 55″·40 to 15′ 53″·68, a reduction of 1″·72, so that, finally, the *N.A.* geocentric mean distance eclipse semidiameter of 15′ 32″·65 must be reduced to 15′ 30″·97, a reduction of 1″·68, corresponding to 3″·36 for the diameter. The Sun's apparent semidiameter at totality comes out as 15′ 46″·59 by *N.A.* data, and is considered to be accurate.

In a paper by Mr. Dyson, giving an account of the total eclipse of 1901 May 18 (*Proc. R.S.* vol. lxix.), it is stated that the observed duration of totality at Aoer Gadang, Sumatra, was 10^s less than that predicted by the *N.A.*, but agreed closely with a prediction based on a value of 15′ 31″·47 for the geocentric mean semidiameter of the Moon. This value, Dr. Downing informs me, was arrived at by Herr Peters (of the Berlin *Jahrbuch*) after a discussion of the eclipse of 1900 May 28.

If we adopt this value for the Navalmoral eclipse, totality

should have lasted $82^{s}\cdot4$. The loss of $2^{s}\cdot4$, needed to produce agreement with our observations, could be accounted for by a reduction of only $0''\cdot5$ in the Moon's semidiameter, *i.e.* of $1''$ in her diameter. This discrepancy may reasonably be attributed to irregularities in the Moon's limb, as suggested by Mr. Crommelin in *Monthly Notices*, vol. lxi. pp. 52–3. A lunar valley of a depth of about 6,000 feet below the Moon's mean level, or two valleys, the sum of whose depths amounts to 6,000 feet, one valley being near second and the other near third contact, would be sufficient.

As the points of second and third contact were about position angles 93° and 271°, and the Moon's north pole at the time of the eclipse was 11° 13′ W. of her north point, the position angles of contact on the Moon's limb would be about 104° and 282° from her north pole. The Moon's libration in latitude was practically zero, that in longitude was +4°. The position angles may easily be 1° or 2° in error, for an extremely slight deviation from central transit produces a considerable change in these angles. The angles 99° and 267°, given by the *Nautical Almanac*, would apparently correspond to a difference of some $2''$ between the centres of the Sun and Moon about mid-totality.

Adopting the angles 104° and 282°, second contact of the Moon with the Sun would occur near 86° E. longitude and 14° S. latitude, whilst third contact would be near 94° W. longitude and 12° N. latitude. The second contact indicates a region between the Cordilleras and the D'Alembert Mountains. The third contact is not on the lunar maps, being on that face of the Moon not shown by them, but the region indicated is in the neighbourhood (N.W.) of Neper, a walled plain.

Mr. Saunder, to whom I wrote about these localities, remarks that it is not possible, from our present knowledge, to say whether the neighbourhood of the Moon's limb near the contact points is above or below the mean level. But he adds that this information might be obtained from photographs taken during the eclipse, or taken at times when the Moon was under libration, similar to that which she had during the eclipse.

It is well known that the lunar border in the vicinity of second contact was seen to be very irregular during the progress of this eclipse. It will be of interest for some one possessed of suitable lunar photographs to investigate the regions of the Moon's limb referred to above.

Dr. Struve (*Monthly Notices*, vol. lii. p. 283) states that there is no systematic deviation of the form of the Moon's disc from a circle, and Professor Franz (*Monthly Notices*, vol. lxi. p. 264) concludes that the Moon's figure is sensibly spherical. Thus surface irregularities seem to be the only thing left to account for the discrepancy, still outstanding at Navalmoral, when Herr Peters's value of the semidiameter is used.

The estimated value of the Moon's telescopic mean semidiameter has undergone some interesting changes. According to the late Mr. Stone (*Monthly Notices*, vol. xlii. p. 42) the

adopted value for 1750–1815 was 15′ 34″·27 ; for 1816–39 it was 15′ 33″·95 ; for 1840–61 it was 15′ 34″·68 ; and in and after 1862 it was 15′ 34″·08. The present (1900) value used in the *Nautical Almanac* is 15′ 34″·09, the mean horizontal parallax being 57′ 2″·28.

In the Preface to the *Nautical Almanac* for 1897 there appeared, for the first time, special values of the semidiameters of the Sun and Moon for use in eclipses and occultations, the value for the Sun being 15′ 59″·63 (Auwers), and for the Moon 15′ 32″·65 (Struve). For 1900 the American Ephemeris adopts 15′ 31″·76 for the eclipse semidiameter of the Moon.

There seems to be but little doubt that we need to use *three* different values for the Moon's semidiameter ; the largest value for ordinary telescopic purposes, the intermediate for the occultations of stars, and the smallest for solar eclipses.

I would draw the attention of those interested in this subject to a very able article by Professor Oudemans, published in the *Monthly Notices*, vol. xxvi. pp. 249–260, 1866 May. He therein discusses the duration of occultations and eclipses, refers to the effect of indentations on the Moon's limb, and draws the conclusion that for occultations the Moon's mean semidiameter is 15′ 32″·27, but that for eclipses it is only 15′ 31″·02. The difference 1″·25 between these values agrees very nearly with the difference 1″·18 between the *Nautical Almanac* value, 15′ 32″·65, and that recently deduced by Herr Peters, 15′ 31″·47.

I am much indebted to Mr. Crommelin for independent confirmation of the principal computations in this paper.

Leeds : 1901 *December.*

Mean Areas and Heliographic Latitudes of Sun Spots in the Year 1901, *deduced from Photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn (India), and in Mauritius.*

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those printed in the *Monthly Notices*, vol. lxi. p. 533, and are deduced from the measurements of photographs taken at the Royal Observatory, Greenwich, at Dehra Dûn, India, and at the Royal Alfred Observatory, Mauritius.

Table I. gives the mean daily areas of umbræ, whole spots, and faculæ for each synodic rotation of the Sun in 1901 ; and Table II. gives the same particulars for the entire year 1901 and the twelve preceding years for the sake of comparison. The areas are given in two forms : first, projected areas, that is to

say, as seen and measured on the photographs, these being expressed as millionths of the Sun's apparent disc; and next areas as corrected for foreshortening, the areas in this case being expressed in millionths of the Sun's visible hemisphere.

Table III. exhibits for each rotation in 1901 the mean daily area of the whole spots, and the mean heliographic latitude of the spotted area, for spots north and for spots south of the equator; together with the mean heliographic latitude of the entire spotted area, and the mean distance from the equator of all spots; and Table IV. gives the same information for the year as a whole, similar results from 1889 to 1900 being added, as in the case of Table II. Tables II. and IV. are thus in continuation of the similar tables for the years 1874 to 1888 on pp. 381 and 382 of vol. xlix. of the *Monthly Notices*.

The rotations in Table I. and Table III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill* by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node at mean noon of 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days. The dates of the commencement of the rotations are given in Greenwich civil time, reckoning from mean midnight.

Table I.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
	1900. d							
632	Dec. 24·64	26	0·0	0·5	0·0	0·0	0·3	0·0
	1901.							
633	Jan. 20·98	27	2·4	8·0	13	1·9	6·6	16
634	Feb. 17·32	26	6·9	22	29	5·0	16	36
635	Mar. 16·65	26	0 0	0·0	7·0	0·0	0·0	9·8
636	Apr. 12·93	27	0·0	0·0	0·0	0·0	0·0	0·0
637	May 10·18	28	138	339	53	83	231	72
638	June 6·39	27	14	70	34	8·8	46	42
639	July 3·59	27	0·0	0·9	47	0·0	1·1	54
640	July 30 80	25	0·0	0·0	6·2	0·0	0·0	6·6
641	Aug. 27·03	28	0·0	0·4	2·2	0·0	0·3	3·2
642	Sept. 23·29	27	2·5	10	8·1	1·4	5·4	7·1
643	Oct. 20·58	27	5·2	29	94	5·5	31	130
644	Nov. 16·88	27	9·7	51	4·0	6·4	35	7·7

TABLE II.

Year.	No. of Days on which Photographs were taken.	Mean of Daily Areas. Projected. Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening. Umbræ.	Whole Spots.	Faculæ.
1889	360	17·9	103	107	13·1	78·0	131
1890	361	21·3	133	273	15·5	99·4	304
1891	363	120	745	1322	86·2	569	1412
1892	362	255	1596	3230	186	1214	3270
1893	362	327	1983	2287	234	1464	2404
1894	364	317	1728	1666	231	1282	1877
1895	364	237	1330	2059	169	974	2278
1896	364	127	745	1243	90	543	1410
1897	364	122	695	977	88	514	1149
1898	363	93	532	767	64	375	891
1899	364	27	159	297	18	111	337
1900	360	22	101	150	17	75	180
1901	359	14	41	23	8·6	29	29

TABLE III.

No. of Rotation.	Date of Commencement of each Rotation.	No. of Days on which Photographs were taken.	Spots North of the Equator. Mean of Daily Areas.	Mean Heliographic Latitude.	Spots South of the Equator. Mean of Daily Areas.	Mean Heliographic Latitude.	Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
	1900. d			°		°	°	°
632	Dec. 24·64	26	0·0	...	0·3	10·20	−10·20	10·20
	1901.							
633	Jan. 20·98	27	0·0	...	6·6	8·20	− 8·20	8·20
634	Feb. 17·32	26	15	3·71	1·2	4·95	+ 3·04	3·81
635	Mar. 16·65	26	0·0	...	0·0	...	...	...
636	Apr. 12·93	27	0·0	...	0·0	...	...	...
637	May 10·18	28	231	8·66	0·0	...	+ 8·66	8·66
638	June 6·39	27	38	9·69	8·6	19·48	− 4·30	11·55
639	July 3·59	27	0·1	2·10	1·0	1·68	− 1·17	1·73
640	July 30·80	25	0·0	...	0·0	...	...	...
641	Aug. 27·03	28	0·3	36·37	0·0	...	+36·37	36·37
642	Sept. 23·29	27	0·0	...	5·4	8·85	− 8·85	8·85
643	Oct. 20·58	27	0·0	...	31	14·91	−14·91	14·91
634	Nov. 16·88	27	0·0	...	35·0	20·21	−20·21	20·21

TABLE IV.

Year.	No. of Days on which Photographs were taken.	Spots North of the Equator. Mean of Daily Areas.	Spots North of the Equator. Mean Heliographic Latitude.	Spots South of the Equator. Mean of Daily Areas.	Spots South of the Equator. Mean Heliographic Latitude	Mean Heliographic Latitude of Entire Spotted Area.	Mean Distance from Equator of all Spots.
			°		°	°	°
1889	360	5·0	7·26	73·0	11·90	−10·68	11·61
1890	361	53·1	22·20	46·3	21·75	+ 1·73	21·99
1891	363	401	20·49	169	19·91	+ 8·52	20·31
1892	362	607	15·09	607	21·69	− 3·29	18·39
1893	360	517	14·91	941	14·26	− 3·93	14·49
1894	364	543	12·31	739	15·56	− 3·75	14·18
1895	364	565	14·26	409	12·54	+ 3·01	13·54
1896	364	203	13·60	340	14·77	− 4·15	14·33
1897	364	196	8·32	318	7·73	− 1·62	7·96
1898	363	110	9·82	266	10·77	− 4·75	10·49
1899	364	23	6·18	88	10·43	− 6·95	9·54
1900	360	26	6·61	49	8·34	− 3·12	7·74
1901	359	22	8·59	6·6	16·27	+ 2·82	10·37

The principal features of the record for 1901 are :—

1. The decline in area of umbræ, whole spots, and faculæ has still continued, and has been most marked ; the relative decrease in mean daily spotted area for 1901 as compared with 1900 being greater than for 1900 as compared with 1899.

2. The decrease in the area of the umbræ has been proportionally less than that for the whole spots.

3. The decrease in the area of the faculæ on the other hand has been very remarkable. The area registered—twenty-nine millionths—is by far the lowest for any year of the Greenwich solar record, the next lowest being ninety millionths for 1878.

4. Except for a single outburst the northern hemisphere has been practically quiescent for the entire year. But that outburst was so much the greatest that it contributed, in its two appearances, 74 per cent. of the entire spotted area of the year, and in consequence of it the mean daily area for the northern hemisphere was more than three times as great as for the southern.

5. Comparing 1901 with 1900 the decrease for the northern hemisphere was only 15 per cent., whilst for the southern it was 86 per cent. The decrease, therefore, has been almost entirely in the southern hemisphere.

6. The only notable outburst of the year was the group referred to above. This was seen first on the east limb on May 19, the morning after the total solar eclipse of May 18. At the times of the eclipse as seen in Mauritius and Sumatra

respectively the centre of the group was 93° and 92° distant from the central meridian, and therefore only just out of sight behind the east limb. The group was situated in N. lat. 9°, and a fine prominence is seen in the Mauritius photographs of the eclipse, rising from the limb in N. lat. 6°, *i.e.* just over the southern end of the spot group. A comparison of the Mauritius and Sumatra eclipse photographs shows that a considerable change has taken place in the form of this prominence in the interval of about 100 minutes which separated the two series of observations. To the north of the prominence, and therefore directly over the spot group itself, the corona is seen on both series of photographs as much disturbed.

The spot group appeared a second time on the east limb on June 16, but had diminished in size, almost to disappearance, before it reached the west limb on June 26.

7. The distribution of the spots in latitude in 1901 shows some tendency in the mean to seek a higher latitude than in 1900, four groups, two in each hemisphere, appearing in latitudes above 20°. But the relatively great area of the group of May 19, in N. lat. 9°, brings the mean distance from the equator of all spots down to 10°·37. This shows a distinct rise in latitude over 1900, when the mean distance was 7°·74. In the closing years of the preceding cycle the mean distance for 1888 was 7°·39, and for 1889, the year of minimum, 11°·61, whilst for 1890, the first year of increased spotted area, it was 21°·99.

8. The number of days without spots has very greatly increased, there having been no fewer than 289 such days out of the 359 days for which there are photographs available. On 277 days neither spots nor faculæ were shown on the photographs. The days for which the photographic record is deficient are January 13, January 24, February 27, April 3, August 3, and August 5.

9. The mean daily spotted area—twenty-nine millionths—for 1901 is much smaller than that for 1889, the year of the last minimum, which was seventy-eight; but it has not fallen quite so low as at the preceding minimum of 1878, when it was only twenty-four. The very slight activity in 1901, and the very large proportion of days on which no spots were seen, together with the appearance of four groups in latitude 20° or above, point to 1901 being probably the year of minimum. These four high latitude groups may, indeed, indicate the commencement of the revival, a view which receives some confirmation from the appearance on March 3 of the present year of two groups, one of them of considerable size, in N. lat. 25°.

1902 *March* 11.

Results of Micrometer Measures of Double Stars made with the 28-inch Refractor at the Royal Observatory, Greenwich, in the year 1901.

(*Communicated by the Astronomer Royal.*)

The measures were made with a bifilar position-micrometer on the 28-inch refractor, aperture 28 inches, focal length 28 feet. The power generally used was 670, but when the definition permitted a power of 1120 was employed for observing very close pairs. A blue glass shade was employed to diminish the light and irradiation when bright stars were observed. The observations were made in variously coloured fields or in a dark field with illuminated wires. The initials in the last column are those of the observers, viz. :—

B. Mr. Bryant. L. Mr. Lewis. W.B. Mr. Bowyer.

Micrometric Observations of Double Stars at the Royal Observatory, Greenwich.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle °	Distance. ″	No. of Nights.	Mags.		Epoch 1901.	Obs.
Σ 3062 ...	0 1	32 7	345·0	1·46	1	6·9	8·0	·753	B.
β 253	0 5	32 2	42·5	0·52	2	8·3	8·4	·436	B.
β 485	0 6	31 48	304·9	0·41	1	8·7	8·8	·753	B.
Krueger 1 ...	0 6	32 43	186·4	1·92	1	9·2	9·5	·753	B.
β 1026 ...	0 7	36 56	329·2	0·29	1	8·1	8·9	·753	B.
β 864	0 8	55 13	283·9	1·14	1	8·4	11·5	·036	L.
OΣ 2	0 8	63 34	37·4	0·68	1	6·5	8·0	·044	L.
			39·0	0·54	2	...		·853	W.B.
			36·0	0·60	2	...		·896	B.
Σ 19	0 12	53 55	133·9	2·06	2	7·0	9·5	·022	L.
			135·5	2·22	1	...		·890	W.B.
β 1093 ...	0 15	79 34	70·7	0·28	1	7·3	8·2	·830	B.
β 1015 ...	0 16	78 14	113·9	0·49	1	8·4	8·6	·835	W.B.
OΣ 7 B.C.	0 16	24 5	107·4	0·54	1	9·0	10·0	·753	B.
β 1156 ...	0 21	26 7	34·5	0·45	1	9·2	9·3	·753	B.
OΣ 12	0 26	36 2	147·1	0·42	1	5·6	5·9	·082	L.
			149·5	0 55	2	...		·436	B.
OΣ 15	0 30	41 32	311·4	0·16	2	7·5	8·5	·436	B.
β 257	0 35	43 17	229·9	0·46	1	8·1	8·8	·753	B.
OΣ 18	0 37	86 23	128·4	1·07	1	7·4	9·5	·835	W.B.
β 865	0 38	47 18	198·5	1·40	1	8·3	8·8	·753	B.
β 866	0 40	47 18	69·1	1·70	1	9·0	9·1	·753	B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1901.	Obs.
	h m	° ′	°	″					
β 495	0 42	71 51	223·6	0·49	3	7·5	7·7	·850	W.B.
Σ 60	0 43	32 43	220·4	...	1	4·0	7·6	·082	L.
(η Cassiopeiæ)			223·5	5·40	1	...		·753	B.
OΣ 20	0 49	71 22	319·6	0·45	2	6·0	6·4	·896	W.B.
Σ 73	0 49	66 55	21·5	0·91	4	6·2	6·8	·622	W.B.
β 1099 ...	0 51	30 11	324·3	0·11	2	6·2	6·2	·436	B.
β 302	0 53	69 9	106·0	0·56	3	6·8	8·0	·300	W.B.
β 1228 ...	1 0	77 13	276·8	0·64	2	8·3	8·9	·896	W.B.
β 303	1 4	66 44	286·3	0·60	1	7·2	7·2	·025	W.B.
β 235	1 5	39 30	93·9	0·64	1	7·0	7·5	·118	B.
Σ 113	1 15	91 2	352·4	1·56	1	6·2	7·2	·830	B.
β 4	1 18	79 9	66 9	0·33	1	7·8	8·8	·830	B.
β 1164 ...	1 22	85 10	157·7	0·26	1	6·7	7·0	·830	B.
β 506	1 26	75 10	16·4	1·25	1	4·0	10·5	·865	L.
(η Piscium)									
Σ 158	1 41	57 20	261·4	1·95	2	8·3	8·8	·032	W.B.
β 509	1 38	80 56	253·2	0·65	1	8·4	8·7	·830	B.
β 1016 ...	1 42	57 25	22·7	0·47	1	8·7	8·7	·923	W.B.
Ho. 311 ...	1 46	45 38	176·1	0·36	3	7·5	7·5	·923	W.B.
Σ 183 A.B.	1 49	61 41	354·6	0·38	4	7·5	8·2	·499	W.B.
A.C.		...	163·5	5·42	2	7·5	8·9	·940	W.B.
Σ 186	1 51	88 39	204·3	0·39	2	7·2	7·2	·819	B.
OΣ 38	1 58	48 10	112·0	0·32	1	5·0	6·2	·053	L.
(γ^2 Andromedæ)			117·1	0·27	1	...		·118	B.
Σ 208	1 58	64 33	71 8	0·61	3	6 2	8·4	·653	W.B.
Σ 228	2 8	42 59	88·6	0·40	1	6·7	7·6	·118	B.
Σ 231	2 8	92 52	231·4	16·24	1	6·0	7·8	·098	L.
			232·0	15·95	1	...		·112	W.B.
β 519	2 25	92 42	37·0	0·55	1	8·3	8·9	·098	L.
OΣ 44	2 36	47 44	232·3	1·23	1	7·8	8·5	·923	W.B.
β 262	2 42	59 22	63·9	1·62	3	8·2	9·6	·369	W.B.
Σ 305	2 42	71 2	314·7	3·01	2	7·3	8·0	·497	W.B.
β 524	2 47	52 5	203·6	0·22	1	5·5	6·5	·008	L.
			17·88	0·13	1	...		·118	B.
Σ 333	2 53	69 4	201·6	1·02	2	5·7	6·0	·021	W.B.
			200·2	1·55	1	...		·865	L.
β 525	2 53	68 47	144·1	0·31	3	7·5	7·5	·621	L.
β 1173 ...	2 53	66 16	330·8	0·34	1	7·7	8·8	·986	L.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1901.	Obs.
Σ 346	2 59	65 8	94·1	0·49	2	6·0 6·0	·040	W.B.
			94·0	0·43	1	...	·808	B.
*	2 59	65 9	166·6	1·93	1	...	·913	L.
β 83	2 41	95 23	103·3	0·92	1	7·9 9·4	·830	B.
β 1030 ...	3 4	68 39	155·2	0·49	1	8·5 8·5	·077	W.B.
			164·6	0·56	2	...	·524	B.
			159·3	0·49	1	...	·986	L.
β 530	3 8	67 25	188·6	1·77	1	9·7 10·1	·986	L.
β 84	3 11	96 17	27·4	0·75	1	6·8 7·3	·830	B.
OΣ 53 ...	3 11	51 44	232·0	0·54	1	7·2 8·0	·923	W.B.
β 878	3 23	67 32	75·0	1·37	1	5·8 13·0	·986	L.
Σ 412	3 29	65 52	19·9	0·30	1	6·6 6·9	·077	W.B.
			6·6	0·24	3	...	·622	B.
β 533	3 29	58 39	44·3	0·48	2	8·0 8·0	·058	W.B.
			53·3	0·49	1	...	·808	B.
Σ 425	3 33	56 10	268·3	2·37	2	7·3 7·3	·021	W.B.
β 536	3 40	66 6	293·2	0·19	1	8·2 8·7	·939	B.
			319·2	0·30	1	...	·986	L.
β 538	3 42	66 12	135·2	2·06	1	10·0 10·5	·986	L.
β 1184... ...	3 42	67 56	272·6	0·39	1	8·0 8·5	·923	W.B.
			271·3	0·58	1	...	·939	B.
OΣ 516 ...	3 44	58 1	42·4	2·09	1	7·2 9·2	·923	W.B.
OΣ 531 ...	4 1	52 12	129·8	1·99	1	8·5 10·2	·112	W.B.
Ho. 326 ...	4 3	61 37	180·8	0·32	1	8·5 8·5	·939	B.
OΣ 77	4 10	58 33	178·0	0·22	1	7·5 7·5	·118	B.
OΣ 80	4 16	47 49	183·2	0·45	1	6·5 7·0	·112	W.B.
			180·1	0·51	2	...	·452	B.
OΣ 74	4 7	80 36	288·4	0·36	1	8·0 8·5	·939	B.
Σ 535	4 18	78 51	320·2	1·70	2	6·7 8·2	·517	W.B.
Aldebaran ...	4 30	73 42	110·0	30·54	1	1 13·5	·036	L.
OΣ 86	4 31	70 27	44·6	0·53	1	7·5 7·5	·099	L.
			61·3	0·43	3	...	·667	W.B.
			55·3	0·43	1	...	·118	B.
*	4 30	70 15	190·4	0·39	1	7·5 8·0	·099	L.
			192·1	0·41	1	...	·121	W.B.
Σ 567	4 31	70 43	319·1	1·70	1	8·5 9·0	·099	L.
			318·5	1·82	2	...	·039	W.B.
Σ 572	4 32	63 18	22·0	3·81	1	6·5 6·5	·077	W.B.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1901.	Obs.
β 883 ...A.B.	4 46	79 6	68·8	0″·38	3	7·5	7·8	·612	L.
			64·2	0·26	3		...	·384	B.
			59·7	0·31	2		...	·117	W.B.
A.C.		...	156·0	18·10	1	7·5	13·0	·099	L.
β 1047...A.B.	5 3	62 5	48·6	0·26	1	7·2	8·8	·246	L.
Σ 645 ...A.C.		...	27·3	11·62	1	7·2	9·2	·246	L.
β 885	5 6	91 54	25·1	0·56	1	8·3	8·4	·118	B.
β 555 ...B.C. (Rigel)	5 10	98 19	179·3	0·36	1	8	8	·036	L.
Σ 668 ...A.B.		...	201·7	9·70	1	1	8	·036	L.
Dawes 5 ...	5 19	92 29	79·8	1·08	1	3·5	5·5	·085	B.
β 889 ...A.B.	5 21	55 40	220·7	1·02	1	8·5	8·8	·246	L.
A.C.		...	98·3	3·62	1		14·0	·246	L.
A.D.		...	108·6	12·58	1		13·8	·246	L.
Σ 707 ...A.E.		...	131·6	18·40	1		9·4	·246	L.
A.F.		...	189·3	27·78	1		10·1	·246	L.
Σ 749	5 31	63 8	168·8	0·74	2	7·1	7·2	·137	W.B.
β 1240...A.B.	5 32	59 34	336·4	0·22	2	5·6	6·0	·124	B.
			331·1	0·18	2		...	·281	L.
A.C.		...	267·4	12·24	1	5·6	8·7	·268	L.
OΣ 112 ...	5 33	52 6	73·6	0·61	1	7·5	7·5	·151	B.
			66·0	0·62	3		...	·287	L.
β 1032 ...	5 34	92 39	328·4	0·31	1	4·0	5·0	·173	B.
OΣ 119 ...	5 43	82 4	323·2	0·72	1	7·5	8·3	·085	B.
β 560	5 43	60 18	166·7	0·69	2	8 0	8·5	·091	B.
Σ 799	5 43	51 28	180·7	0·80	2	7·2	8·3	·308	L.
Aitken 54 A.B.	6 3	60 45	347·0	0·58	2	7·6	8·5	·270	L.
A.C.		...	312·1	51·86	1		...	·296	L.
β 1058... ...	6 4	66 59	264·0	0·31	2	7·2	7·5	·091	B.
			273·1	0·27	1		...	·293	L.
Σ 899	6 17	72 22	20·5	2 26	2	7·0	8·0	·140	W.B.
β 1021 ...	6 25	61 32	82·5	0·76	2	8 1	9·4	·285	L.
OΣ 149 ...	6 30	62 38	280·0	0·72	2	6·5	9·0	·280	L.
Σ 945	6 33	48 56	267·4	0·72	2	7·1	8·0	·265	L.
			263·5	0·83	1		...	·309	W.B.
OΣ 156 ...	6 42	71 42	301·9	0·67	3	6·5	7·0	·111	B.
			310·4	0·58	1		...	·233	W.B.
Σ 1037 ...	7 7	62 35	301·0	0·89	2	7·0	7·1	·155	L.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1901.	Obs.
	h m	° ′	°	″			·	
Σ 1066 ...	7 14	67 50	211·0	7·44	1	3·2 8·2	·233	W.B.
β 579	7 27	56 40	28·7	0·96	1	8·0 10·5	·323	L.
Σ 1110 ...	7 28	57 54	226·0	5·62	1	2·7 3·7	·044	L.
(Castor)			223·6	5·75	2	...	·154	W.B.
OΣ 175 ...	7 28	58 49	338·4	0·75	2	6·0 6·6	·174	B.
			332·6	0·63	2	...	·230	W.B.
			329·9	0·61	1	...	·323	L.
OΣ 177 ...	7 35	52 20	127·6	0·61	1	7·5 8·5	·233	W.B.
			122·7	0·58	2	...	·388	B.
			125·9	0·56	2	...	·308	L.
β 580	7 39	61 44	137·1	1·45	1	2·0 14·3	·312	L.
(Pollux C.D)								
β 101	7 47	103 38	302·7	0·48	1	5·2 6·5	·315	B.
OΣ 185 ...	7 52	88 36	24·1	0·26	1	6·8 7·0	·304	B.
β 581 A.B.	7 59	77 26	291·3	0·28	3	8·5 8·6	·195	B.
			282·0	0·68	1	...	·312	L.
A.C.	...		200·0	5·57	1	11·0	·312	L.
Σ 1184 ...	8 3	51 49	340·8	27·37	1	8·0 8·5	·227	W.B.
			341·0	26·97	1	...	·293	L.
Σ 1186 ...	8 3	62 13	220·6	3·51	1	7·1 10·4	·293	L.
Σ 1187 ...	8 3	57 28	43·1	2·37	3	7·1 8·0	·256	W.B.
Σ 1196 A.B.	8 6	72 3	5·6	0·84	1	5·0 5·7	·176	L.
ζ Cancri			2·2	1·08	2	...	·270	W.B.
A.C.	...		113·0	5·44	1	5·0 6·5	·176	L.
			114·3	5·51	2	...	·270	W.B.
B.C.	...		124·1	6·12	1	5·7 6·5	·176	L.
			126·3	6·20	2	...	·270	W.B.
Σ 1202 ...	8 8	78 51	318·2	2·43	1	7·7 9·8	·296	W.B.
β 1244 ...	8 9	87 43	43·8	0·70	2	7·9 8·1	·239	B.
Σ 1211 ...	8 12	50 42	133·3	1·17	1	8·7 9·2	·233	W.B.
Σ 1216 ...	8 16	91 17	202·0	0·47	1	7·5 8·2	·304	B.
β 585	8 36	69 10	111·4	0·41	1	7·5 9·0	·255	W.B.
			116·7	0·37	2	...	·315	B.
β 209	8 37	50 50	357·8	1·72	1	8·2 8·7	·233	W.B.
Σ 1273 ...	8 41	83 13	232·3	3·51	2	3·5 7·7	·269	L.
ε Hydræ A.C.								
Schiaparelli A.B.	...		280·6	0·11	2	3·5 6·0	·284	L.
			272·3	0·18	3	...	·306	B.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1901.	Obs.
β 1068 ...	8 44	80 46	172·7	0·45	1	7·7 8·8	·306	L.
			178·9	0·36	1	...	·315	B.
*	8 45	81 21	43·9	3·06	1	...	·277	W.B.
Perrotin ...	8 46	81 17	338·5	0·71	2	7·5 8·7	·306	B.
			349·7	0·53	1	...	·306	L.
β 587	8 47	96 48	137·6	0·73	1	6·0 9·0	·315	B.
Σ 1321 ...	9 8	36 52	65·3	19·05	1	7·1 7·2	·298	B.
Σ 3121 ...	9 12	61 00	17·9	0·77	1	7·5 7·8	·085	B.
			21·6	0·66	2	...	·272	W.B.
			20·6	0·84	1	...	·294	L.
OΣ 201 ...	9 18	61 39	221·8	1·39	3	7·5 8·9	·287	W.B.
			218·1	1·43	1	...	·293	L.
β 105	9 19	63 2	200·6	2·58	1	5 10·5	·293	L.
*	9 19	63 33	17·6	3·40	1	9·5 10·0	·293	L.
Σ 1348... ...	9 19	83 14	319·9	1·86	2	7·5 7·6	·287	W.B.
			319·9	1·98	1	...	·290	L.
Aitken ...	9 20	58 25	46·0	0·90	1	8·7 10·2	·323	L.
Σ 1356 ...	9 23	80 27	111·6	0·64	3	6·2 7·0	·291	B.
			112·1	...	1	...	·361	L.
Σ 1374... ...	9 35	50 35	284·5	...	1	7·0 8·3	·364	W.B.
Σ 1385... ...	9 44	72 55	182·4	1·19	1	8·5 10·7	·312	L.
OΣ 208 ...	9 45	35 28	290·8	0·32	4	5·0 5·6	·297	B.
Σ 1389... ...	9 47	62 33	306·3	2·32	3	8·0 9·0	·294	W.B.
			313·0	2·15	1	...	·312	L.
A.C. 5... ...	9 48	97 36	89·7	0·35	2	5·5 5·7	·307	B.
OΣ 215 ...	10 11	71 45	211·5	0·93	2	6·7 7·0	·222	B.
			206·3	0·88	3	...	·316	W.B.
			212·4	1·01	2	...	·317	L.
Σ 1424 ... γ Leonis	10 14	69 38	114·1	3·77	4	2·0 3·5	·299	W.B.
			116·9	...	1	...	·359	L.
Σ 1426 A.B.	10 15	83 4	283·4	0·76	1	7·8 8·3	·296	W.B.
			280·1	0·77	2	...	·321	B.
			291·7	...	1	...	·361	L.
A.C.	...		6·2	7·54	1	7·8 9·3	·296	W.B.
			12·2	...	1	...	·361	L.
OΣ 216 ...	10 17	74 9	123·2	1·40	1	7·0 10·5	·312	L.
Σ 1429 ...	10 19	64 53	249·7	0·99	4	8·1 8·5	·305	W.B.
			248·2	0·65	1	...	·323	L.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1901.	Obs.
OΣ 217 ...	10 21	72 16	148·8	0·87	2	7·3 7·8	·222	B.
			145·1	0·88	3	...	·310	W.B.
OΣ 218 ...	10 22	85 56	75·5	0·82	2	7·3 9·2	·321	B.
Σ 1457 ...	10 33	83 45	316·6	1·35	1	7·4 8·4	·296	W.B.
			314·1	...	1	...	·361	L.
OΣ 224 ...	10 34	80 38	296·5	0·39	2	7·2 9·2	·321	B.
OΣ 225 A.B.	10 35	70 14	243·3	0·92	2	7·5 11·2	·307	L.
A.C.			351·8	6·30	2	7·5 9·8	·307	L.
OΣ 227 ...	10 36	78 44	348·5	0·61	2	7·5 8·5	·293	B.
			345·2	0·65	2	...	·306	L.
OΣ 228 ...	10 42	66 54	188·3	0·58	2	7·2 8·1	·222	B.
			185·2	0·41	1	...	·288	W.B.
			191·9	0·57	2	...	·318	L.
OΣ 229 ...	10 42	48 21	318·4	0·85	4	6·7 7·1	·256	B.
β 915	10 44	65 11	230·1	1·57	1	9·0 9·2	·312	L.
			227·2	1·42	1	...	·315	B.
Ho. 378 ...	10 59	51 2	225·3	0·48	3	8 8·1	·289	B.
Ho. 48... ...	10 59	66 18	12·1	1·46	1	8·3 11	·236	L.
Σ 1517 ...	11 8	69 19	267·4	0·52	3	7·3 7·3	·233	B.
			265·8	0·44	1	...	·312	L.
			272·5	...	1	...	·364	W.B.
Σ 1523 ... (ζ Ursæ Maj.)	11 13	57 54	147·5	2·29	1	4·0 4·9	·323	L.
			146·8	2·28	4	...	·342	W.B.
Σ 1527 ...	11 14	75 11	16·6	3·01	2	6·9 3·1	·370	W.B.
			17·4	3·40	2	...	·374	L.
Σ 1536 ...	11 19	78 55	55·7	2·43	2	3·9 7·1	·370	W.B.
			49·7	2·39	2	...	·372	L.
Lalande 21846	11 24	58 59	9·5	0·69	1	7 11	·288	W.B.
			8·8	0·92	3	...	·290	L.
OΣ 234 ...	11 25	48 10	144·1	0·26	3	7·0 7·4	·352	B.
			137·9	0·27	1	...	·504	L.
OΣ 235 ...	11 26	28 22	121·5	0·42	1	6·0 7·3	·501	B.
			124·7	0·47	1	...	·504	L.
Σ 1554 ...	11 31	76 36	250·4	0·78	1	8·9 9·1	·315	B.
			251·9	0·96	1	...	·383	L.
Σ 1555 A.B.	11 31	61 40	350·3	0·32	2	6·4 6·8	·272	L.
			352·1	0·39	4	...	·301	B.
Σ 1555 A.C.	11 31	61 40	146·7	20·79	1	6·4 10·3	·288	W.B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1901.	Obs.
	h m	° ′	°	″				
OΣ 237 ...	11 34	48 18	263·4	1·20	1	7·4 9·0	·504	L.
Σ 1586 ...	11 52	49 6	249·0	1·48	1	8·2 10·2	·504	L.
Σ 1606 ...	12 6	49 33	331·3	0·97	1	6·3 7·0	·392	B.
			330·3	0·97	2	...	·510	L.
Σ 1639 ...	12 19	63 52	357·7	0·30	3	6·5 8·0	·289	L.
			1·4	0·30	2	...	·321	B.
Σ 1643 ...	12 22	62 25	39·8	2·12	2	8·3 8·6	·292	W.B.
			41·0	1·98	3	...	·360	L.
Σ 1647 ...	12 25	79 44	220·5	1·57	3	7·5 8·0	·345	W.B.
			220·4	1·26	3	...	·378	L.
Σ 1661 ...	12 30	78 3	237·0	2·65	1	8·5 8·5	·288	W.B.
			237·1	2·51	2	...	·386	L.
Σ 1658 ...	12 30	82 0	357·2	2·28	2	8 0 9·8	·386	L.
Σ 1663 ...	12 32	68 15	100·0	0·56	2	7·8 8·7	·292	W.B.
			103·2	0·66	1	...	·315	B.
			105·5	0·67	2	...	·386	L.
Σ 1670 ... (γ Virginis)	12 37	90 54	144·5	5·86	1	3·0 3·0	·375	W.B.
Ho. 256 ...	12 39	53 41	130·6	0·49	1	6·5 9·0	·236	L.
Σ 1687 A.B.	12 48	68 13	82·7	1·27	4	5·1 7·8	·364	W.B.
			83·1	1·19	2	...	·402	L.
Σ 1687 A.C.	12 48	68 13	126·3	28·60	1	5·1 9·0	·421	L.
OΣ 256 ...	12 51	90 25	74·2	0·55	3	7·2 7·6	·313	B.
β 1081... ...	12 55	58 41	343·0	5·26	1	4·5 14·0	·421	L.
β 1082... ...	12 56	33 6	100·9	1·15	1	6·0 9·6	·515	L.
Σ 1711 ...	12 58	75 59	345·9	0·91	2	8·5 9·0	·402	L.
β 929	12 59	93 7	222·8	0·46	2	6·2 6·2	·313	B.
OΣ 260 ...	13 3	62 31	125·3	0·65	2	7·9 8 3	·364	L.
			123·6	...	1	...	·326	B.
			122·3	0·69	1	...	·419	W.B.
Σ 1728 ...	13 5	71 57	196·7	0·31	3	6·0 6·0	·370	L.
			196·6	0·28	1	...	·326	B.
			195·4	0·16	1	...	·419	W.B.
β 930	13 1	44 12	120·2	2·74	1	6·0 11·3	·515	L.
β 1083 A.B.	13 1	60 27	223·0	6·74	1	6·5 11·5	·422	L.
B.C.		...	221·3	0·35	1	11·5 11·7	·422	L.
✶	13 2	62 31	202·9	0·42	1	9·0 9·5	·422	L.

Star's Name.		R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1901.	Obs.
		h m	° ′	°	″				
OΣ 261	...	13 7	57 23	347·0	1·33	1	7·0 7·5	·390	L.
				343·0	1·52	3	...	·434	W.B.
β 800 ...	...	13 12	72 27	117·6	2·66	2	7·1 10·2	·348	L.
Σ 1734	...	13 16	86 32	188·5	...	1	7·2 7·9	·367	L.
Σ 1742	...	13 19	88 5	347·8	...	1	7·4 7·9	·367	L.
Ho. 260	...	13 19	60 15	323·6	0·78	2	8·3 8·5	·364	L.
				314·7	...	1	...	·326	B.
				316·2	0·78	1	...	·419	W.B.
OΣ 266	...	13 24	73 47	337·9	1·65	1	7·3 7·8	·375	W.B.
				391·8	1·92	2	...	·385	L.
OΣ 269	...	13 28	54 34	211·0	0·32	1	6·5 7·0	·392	B.
				218·0	0·38	1	...	·422	L.
				219·4	0·27	2	...	·430	W.B.
Σ 1768	...	13 33	53 13	130·3	1·12	3	5·0 7·6	·448	W.B.
				130·6	0·89	1	...	·534	B.
β 612 ...	...	13 35	78 45	230·4	0·30	1	6·3 6·5	·392	B.
				228·7	0·25	2	...	·463	W.B.
Σ 1785	...	13 45	62 31	285·0	1·26	2	7·2 7·5	·430	W.B.
				280·1	1·40	1	...	·389	L.
β 1270	...	13 59	81 2	15·1	0·31	1	8·2 8·3	·507	W.B.
Σ 1808	...	14 6	62 56	255·3	2·73	1	8·0 9·0	·389	L.
				254·8	2·52	1	...	·485	W.B.
OΣ 278	...	14 8	45 21	88·6	0·31	2	7·5 7·5	·403	B.
				90·2	0·39	1	...	·504	L.
				90·3	0·24	1	...	·507	W.B.
Σ 1819	...	14 10	86 24	177·5	1·33	1	7·9 8·0	·485	W.B.
Σ 1820	...	14 10	34 13	254·5	2·00	1	8·2 8·5	·515	L.
β 1272	A.B.	14 14	40 47	136·1	1·31	3	8·4 9·5	·446	L.
				129·2	1·47	1	...	·507	W.B.
	A.C.			320·5	24·22	1	8·4 8·5	·504	L.
β 1273...	...	14 15	41 37	184·5	1·08	2	8·6 10·8	·332	L.
Σ 1834	...	14 17	41 2	202·0	0·23	1	7·1 7·2	·159	L.
				179·0	0·1	1	...	·504	L.
				219·4	0·18	3	...	·311	B.
Σ 1863	...	14 35	38 0	92·9	0·52	1	7·1 7·4	·507	W.B.
				89·4	0·69	1	...	·515	L.
Σ 1865	...	14 36	75 51	150·9	0·31	2	3·5 3·9	·348	L.
				151·8	0·36	4	3·5 3·9	·394	B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1901.	Obs.
	h m	° ′	°	″				
Σ 1867 ...	14 36	58 17	12·9	1·24	3	7·7 8·2	·452	W.B.
Σ 1871 ...	14 38	38 11	292·3	1·76	1	7·0 7·0	·507	W.B.
			295·2	1·88	1	...	·515	L.
Σ 1877 ...	14 41	62 30	321·7	2·85	1	3·0 6·3	·389	L.
			327·6	2·67	1	...	·419	W.B.
Σ 1879 ...	14 41	79 55	132·7	0·47	2	8·0 8·5	·478	L.
OΣ 285 ...	14 42	47 11	123·2	0·29	3	7·1 7 6	·346	B.
			125·4	0·33	1	...	·507	W.B.
Σ 1883 ...	14 44	83 37	243·8	0·52	2	7·0 7·0	·398	B.
Σ 1888 ...	14 47	70 29	201·4	2·77	2	5·1 7·4	·400	L.
OΣ 287 ...	14 48	44 40	321·7	0·70	4	7·5 7·6	·514	B.
			322·3	0·83	1	...	·507	W.B.
β 31	14 48	70 51	197·4	1·39	1	8·4 9·7	·411	L.
			198·8	1·45	1	...	·496	W.B.
OΣ 288 ...	14 49	73 53	194·4	1·58	1	6·5 7·0	·411	L.
			191·3	1·67	2	...	·491	W.B.
Σ 1908 ...	15 1	55 7	142·2	1·47	2	8·2 9·2	·491	W.B.
Σ 1909 ...	15 1	41 57	63·9	4·31	1	5·2 6·1	·518	W.B.
Ho. 60 ...	15 9	54 44	28·4	0·31	1	8 8	·304	B.
			29·6	0·31	1	...	·507	W.B.
Σ 1932 ...	15 14	62 48	334·4	0·69	2	5·5 6·0	·354	B.
			334·9	0·63	1	...	·411	L.
			328·8	0·50	1	...	·496	W.B.
Hussey 146...	15 16	68 35	176·5	0·41	2	8·7 9·0	·445	L.
Ho. 62... ...	15 17	54 40	282·3	1·14	2	8·2 8·3	·408	L.
			286·8	1·19	1	...	·485	W.B.
Σ 1937 ...	15 19	59 21	9·3	0·72	2	5·0 5·7	·354	B.
			5·7	0·75	2	...	·356	L.
Σ 1938 ...	15 21	52 18	75·1	0·91	2	6·7 7·3	·491	W.B.
			77·5	1·00	1	...	·534	B.
Aitken 82 ...	15 23	65 45	324·6	0·74	3	8·6 9·1	·414	L.
OΣ 296 ...	15 23	45 39	305·4	1·54	1	7·0 8·6	·515	L.
Σ 1957 ...	15 31	76 44	149·1	1·19	2	8·0 9·5	·469	W.B.
			157·2	1·03	1	...	·479	L.
OΣ 298 ...	15 33	49 52	186·5	0·93	2	7·0 7·3	·354	B.
			181·4	0·94	1	...	·516	L.
Σ 1967 ...	15 39	63 23	110·6	0·54	1	4·0 7·0	·411	L.
			118·8	0·46	3	...	·414	B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1901.	Obs.
	h m	° ′	°	″				
β 619	15 39	76 1	8·0	0·39	2	6·5 7·0	·469	B.
			2·6	0·46	1	...	·479	L.
			3·3	0·40	1	...	·496	W.B.
β 621	15 47	44 58	52·1	0·41	1	8·1 9·3	·507	W.B.
			56·5	0·43	1	...	·731	B.
OΣ 303 ...	15 56	76 27	149·0	0·84	1	7·0 8·0	·403	B.
			140·2	0·81	1	...	·441	W.B.
			148·3	0·67	1	...	·479	L.
β 355	16 5	44 21	285·2	0·36	2	7·8 8·9	·704	B.
Σ 2021 ...	16 9	76 12	153·0	3·98	1	6·7 6·9	·441	W.B.
Σ 2032 ...	16 11	55 54	215·0	4·53	1	5·0 6·1	·687	B.
β 951	16 20	56 23	51·0	0·91	1	8·1 9·0	·411	L.
			52·9	0·81	1	...	·539	W.B.
Σ 2054 ...	16 24	28 4	357·1	1·06	...	5·7 6·9	·787	B.
β 813	16 24	63 15	169·2	1·04	1	8·5 8·5	·523	L.
			169·9	1·01	1	...	·594	W.B.
			163·1	1·09	1	...	·676	B.
β 814	16 24	49 53	322·4	0·34	2	8·4 8·7	·704	B.
Σ 2052 ...	16 25	71 23	94·4	1·66	3	7·5 7·5	·492	W.B.
			96·0	1·83	1	...	·556	L.
Σ 2055 ...	16 26	87 48	57·8	1·04	1	4·0 6·1	·545	B.
Σ 3105 ...	16 26	96 49	30·8	0·44	2	7·5 7·5	·469	B.
β 817	16 28	66 33	144·3	1·05	1	8·2 8·2	·523	L.
			147·8	1·21	1	...	·676	B.
Σ 2084 ... (ζHerculis)	16 38	58 13	231·5	0·84	2	3·0 6·5	·364	L.
			222·3	0·99	5	...	·529	L.
			221·0	1·08	4	...	·684	L.
			228·7	0·67	3	...	·479	B.
			227·3	0·82	2	...	·665	B.
			225·4	0·90	2	...	·786	B.
			224·4	0·93	4	...	·586	W.B.
Σ 2091 ...	16 39	48 37	307·0	1·07	2	7·5 8·0	·575	W.B.
			305·8	1·04	2	...	·682	B.
De. 15... ...	16 41	46 20	321·8	0·54	2	8·2 8·6	·704	L.
			315·0	0·48	1	...	·731	B.
OΣ 315 ...	16 46	88 37	158·5	0·67	1	6·2 8·1	·545	B.
β 821	16 48	57 59	315·5	1·41	1	8·4 8·9	·643	W.B.
			309·5	1·38	2	...	·676	L.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1901.	Obs.
	h m	° ′	°	″				
Σ 2107 ...	16 48	61 10	332·6	0·36	2	6·7 8·5	·676	L.
Σ 3107 ...	16 54	85 53	93·4	1·45	1	8·5 8·5	·545	B.
Σ 2118 ...	16 56	24 49	95·5	0·15	3	5·5 6·5	·625	B.
Σ 2114 ...	16 57	81 24	159·5	1·35	1	6·2 7·4	·496	W.B.
			160·2	1·16	1	...	·545	B.
β 357	17 1	79 19	299·3	1·02	1	8·4 9·4	·712	L.
Σ 2130 ...	17 3	35 24	145·5	2·27	1	5·0 5·1	·797	B.
β 1118 ...	17 5	105 36	251·0	0·33	3	3·4 3·9	·571	B.
Σ 2140 ... (α Herculis)	17 10	75 30	112·1	4·87	1	3·0 6·1	·718	W.B.
Σ 2145 ...	17 12	63 17	56·6	0·52	1	8·3 8·9	·411	L.
			45·7	0·63	1	...	·657	L.
OΣ 327 ...	17 12	33 46	313·0	0·20	1	7·6 8·0	·797	B.
β 628	17 15	57 14	346·7	0·57	1	9·0 9·5	·427	L.
β 629	17 15	57 14	344·1	1·15	1	8·4 8·7	·427	L.
β 630	17 17	57 40	227·3	1·58	1	8·5 9·6	·427	L.
Σ 2163 ...	17 20	47 47	90·7	1·64	2	9·2 9·2	·433	L.
β 1249 ...	17 21	36 3	81·6	0·47	1	8·8 8·9	·433	L.
β 1250 ...	17 21	59 9	70·2	1·75	2	9·4 9·5	·433	L.
Σ 2173 ...	17 25	90 59	327·7	0·94	4	6·0 6·3	·627	B.
β 631	17 35	90 36	63·1	0·34	4	7·5 7·6	·630	B.
Σ 2199 ...	17 37	34 12	90·0	1·59	2	7·2 7·8	·433	L.
β 1251 ...	17 38	74 0	65·2	1·24	2	6·0 11·5	·507	L.
Σ 2203 ...	17 38	48 18	317·0	0·67	2	7·5 7·8	·433	L.
			319·7	0·70	1	...	·643	W.B.
Σ 2205 ...	17 40	72 14	305·7	2·24	1	8·3 8·7	·518	W.B.
			307·4	2·15	3	...	·552	L.
Σ 2214 A.B.	17 40	46 13	213·1	19·16	1	8·3 8·8	·430	L.
De. B.C.	17 40	46 13	142·6	1·16	1	8·8 10·3	·430	L.
Σ 2215 ...	17 41	72 15	292·4	0·75	2	5·9 7·9	·529	W.B.
			295·1	0·61	3	...	·552	L.
Ho. 70 ...	17 42	59 25	104·0	0·53	3	8 8	·425	L.
A.C. 7... ...	17 43	62 13	61·7	1·52	2	10·0 10·5	·507	L.
OΣ 337 ...	17 46	82 44	280·4	0·36	3	7·5 8·0	·611	B.
OΣ 338 ...	17 47	74 39	18·1	0·58	2	6·5 7·0	·529	W.B.
			15·1	0·67	1	...	·545	B.
A.C. 8... ...	17 49	60 18	75·6	0·30	1	8·0 9·8	·731	B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1901.	Obs.
	h m	° ′	°	″				
A.C. 9... ...	17 50	60 10	55·8	1·01	3	8·4 8·7	·552	W.B.
			55·8	1·10	1	...	·731	B.
Σ 2262 ...	17 57	98 11	258·6	1·93	1	5·0 5·7	·687	B.
Σ 2272 ... 70 Ophiuchi	18 0	87 28	227·5	1·58	10	4·3 6·3	·598	W.B.
Σ 2275 ...	18 0	50 39	133·1	0·66	3	9·0 9·2	·466	L.
OΣ 341 ...	18 1	68 34	88·3	0·49	1	6·4 7·7	·559	L.
			86·2	0·30	3	...	·654	B.
A.C. 15 ...	18 3	59 27	319·6	1·07	1	5·4 9·0	·482	L.
Σ 2281 ...	18 4	86 1	236·2	0·22	1	6·7 7·2	·633	B.
Σ 2283 ...	18 5	83 52	83·5	0·93	2	7·2 7·7	·540	W.B.
Σ 2289 ...	18 6	73 32	228·8	1·22	4	6·5 7·0	·563	W.B.
Σ 2294 ...	18 9	89 51	86·5	0·18	2	7·4 7·7	·660	B.
β 1091 ...	18 9	51 26	26·4	0·25	1	8·6 8·6	·731	B.
β 641	18 18	68 32	339·8	0·84	2	7·5 9·0	·569	W.B.
β 1203 ...	18 18	89 15	71·2	0·24	2	7·5 7·7	·682	B.
Σ 2315 ...	18 21	62 40	193·1	0·34	4	7·5 9·0	·561	L.
			202·4	0·24	1	...	·539	W.B.
Ho. 84 ...	18 20	62 40	319·5	4·08	3	9·0 10·0	·518	L.
Σ 2318 ...	18 21	64 4	254·1	20·75	1	7·9 10·0	·482	L.
*	18 22	64 2	138·8	1·50	1	9·5 10·0	·482	L.
*	18 23	64 2	263·7	2·94	1	9·5 10·0	·542	L.
OΣ 351 ...	18 23	41 18	23·7	0·59	2	8·0 8·3	·754	B.
OΣ 543 ...	18 23	43 10	133·4	1·08	2	7·2 9·2	·754	B.
OΣ 357 ...	18 31	78 21	239·3	0·39	1	7·5 7·6	·539	W.B.
			247·3	0·43	1	...	·633	B.
OΣ 358 ...	18 31	73 6	193·0	1·87	3	6·8 7·2	·543	W.B.
Σ 2356 ...	18 34	61 23	60·4	0·89	2	8·2 8·6	·619	L.
			56·0	1·15	2	...	·619	W.B.
*	18 34	61 18	253·4	1·11	1	9·0 10·0	·600	W.B.
			251·0	1·14	1	...	·695	L.
Ho. 87 ...	18 34	73 34	103·0	0·45	1	7·8 8·1	·539	W.B.
Ho. 437 A.B.	18 37	58 27	120·9	0·30	2	8·3 8·5	·511	L.
C.D.	...		341·2	3·48	2	11·2 11·7	·511	L.
Σ 2367 ...	18 37	59 48	94·1	0·24	4	7·2 7·6	·568	L.
			76·0	0·23	1	...	·631	B.
Σ 2369 ...	18 39	87 29	96·8	0·97	1	7·5 8·0	·687	B.
Σ 2375 ...	18 41	84 36	113·7	2·37	2	6·2 6·6	·605	W.B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1901.	Obs.
	h m	° ′	°	″				
Σ 2400 A.B.	18 44	73 51	182·9	1·88	1	8·0 10·6	·745	L.
A.C.		...	187·8	3·08	1	8·0 11·0	·745	L.
Σ 2402 ...	18 45	79 26	207·2	0·84	1	8·0 8·4	·638	W.B.
β 265	18 45	78 36	226·3	1·30	1	7·7 9·3	·742	L.
Hussey 199...	18 47	78 19	4·3	0·19	1	8·7 9·1	·745	L.
β 648	18 53	57 14	220·0	1·11	2	6·0 9·5	·608	L.
Σ 2422 ...	18 53	64 2	95·9	0·91	3	7·6 7·7	·614	W.B.
Σ 2424 ...	18 54	76 31	264·6	16·68	1	5·7 9·2	·745	L.
β 649	18 55	57 40	5·1	1·54	1	8·5 10·7	·673	L.
Σ 2437 ...	18 57	70 58	65·4	0·77	3	7·8 8·2	·627	W.B.
β 287	19 1	76 17	63·4	5·76	1	3 13·0	·742	L.
Σ 2454 ...	19 2	59 43	247·0	0·69	1	8·0 9·2	·643	W.B.
Σ 2455 ...	19 3	67 59	80·0	3·43	2	7·2 8·3	·619	W.B.
Ho. 98 ...	19 4	63 4	145·8	0·24	1	8 8	·841	B.
β 1204 ...	19 7	87 33	195·4	0·35	2	7·7 8·5	·709	B.
Σ 2488 ...	19 11	70 9	325·7	1·50	1	8·5 9·7	·638	W.B.
OΣ 368 ...	19 12	74 1	216·1	0·96	2	7·3 8·5	·619	W.B.
OΣ 371 ...	19 12	62 43	158·7	0·83	2	7 7	·628	B.
			150·8	0·84	2	...	·641	W.B.
β 1256 ...	19 14	83 51	39·2	0·71	3	8·2 8·3	·698	B.
β 141	19 18	67 41	90·3	0·81	1	7·5 8·5	·575	L.
			84·6	0·82	1	...	·643	W.B.
*	19 19	67 41	281·3	0·78	1	9·0 10·0	·643	L.
β 1129 ...	19 19	37 49	329·7	0·28	1	6·3 6·3	·797	B.
Σ 2525 ...	19 22	62 52	319·8	0·51	3	7·4 7·6	·650	L.
			313 0	0 55	1	...	·643	W.B.
σ 623	19 27	62 15	54·6	34·69	2	3 7	·704	L.
Σ 2556 ...	19 35	67 58	142·6	0·37	1	7·3 9·8	·731	B.
β 658	19 40	63 7	295·5	0·39	1	6·5 9·5	·731	B.
A. G. C. 11...	19 45	71 6	151·9	0·18	5	5·7 8·8	·712	B.
Ho. 580 ...	19 49	67 48	273·8	0·66	2	8 8·1	·704	B.
β 1258 ...	19 56	60 22	153·5	1·40	2	8·0 10·7	·627	L.
*	19 57	60 20	130·0	3·40	2	9·2 9·8	·627	L.
β 439 ...	19 57	60 25	237·6	3·25	2	7·9 12·7	·627	L.
Σ 2616 ...	19 58	75 42	265·0	3·23	1	6·8 9·7	·769	W.B.
OΣ 395 . .	19 58	65 21	100·4	0·55	1	5·5 6·0	·580	L.
			102·0	0·57	2	...	·578	B.
			103·6	0·64	2	...	·641	W.B.

Star's Name.	R.A. 1900. h m	N.P.D. 1900. ° ′	Position Angle. °	Distance. ″	No. of Nights.	Mags.	Epoch 1901.	Obs.
Σ 2620 ...	19 59	78 30	291·1	1·78	2	8·2 9·3	·742	W.B.
Σ 2624 ...	20 0	54 15	173·2	1·76	1	7·2 7·8	·715	W.B.
β 57	20 1	74 47	118·0	2·37	1	6·7 11·3	·745	L.
Σ 2695 ...	20 28	64 32	80·6	1·11	1	6·2 8·0	·580	L.
			81·8	1·19	2	...	·641	W.B.
β 670	20 28	76 24	39·3	0·51	2	8·5 8·9	·717	L.
*	20 28	76 23	140·7	0·43	2	9·0 9·5	·717	L.
β 151 ... A.B. (β Delphini)	20 33	75 45	14·0	0·54	3	4·5 6·0	·634	L.
			16·8	0·45	8	...	·681	B.
			14·6	0·39	2	...	·641	W.B.
A.C.	...		119·1	25·11	1	...	·832	L.
A.D.	...		330·3	37·80	1	...	·832	L.
Σ 2729 ...	20 46	96 0	234·0	0·26	2	5·9 7·2	·694	B.
β 367	20 51	62 28	133·3	0·40	1	8·0 6·0	·772	L.
Σ 2737 ...	20 54	86 5	285·3	0·62	4	5·7 8·6	·707	B.
OΣ 424 ...	20 55	74 49	313·1	0·42	1	7·5 8·7	·580	L.
			321·2	0·43	2	...	·742	B.
β 69	20 58	68 46	315·2	0·82	1	8·3 9·1	·580	L.
Σ 2744 ...	20 58	88 52	163·0	1·37	3	6·3 7·0	·717	B.
Σ 2749...A.B.	21 0	86 51	155·2	4·13	1	7·7 8·9	·772	L.
A.D.	...	...	153·2	...	1	7·7 8·9	·772	L.
Secchi ...B.C.	...	...	334·8	0·88	1	8·9 10·0	·772	L.
OΣ 527 ...	21 3	85 15	267·4	0·29	2	6·5 8·6	·709	B.
γ Equulei ...	21 5	80 16	273·3	2·33	1	5·0 11·5	·559	L.
Ho. 152 ...	21 8	62 4	326·5	0·43	3	8·5 9·5	·869	W.B.
OΣ 535 ... (δ Equulei)	21 10	80 24	196·2	0·22	1	4·1 4·1	·534	B.
			196·3	0·21	1	...	·622	B.
			179·0	0·24	1	...	·654	B.
			71·7	0·08	1	...	·687	B.
			69·3	0·11	1	...	·701	B.
			64·1	0·11	1	...	·731	B.
			66·3	0·05	1	...	·753	B.
			64·5	0·09	1	...	·764	B.
			54·4	0·06	1	...	·797	B.
			55·1	0·11	1	...	·830	B.
			50·8	0·12	1	...	·841	B.
OΣ 535 ...	21 10	80 24	185·0	0·21	1	...	·543	L.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.	Epoch 1901.	Obs.
	h m	° ′	°	″				
δ Equulei ...	...	...	193·1	0·27	1	...	·559	L.
			193·9	0·22	1	...	·577	L.
			161·2	...	1	...	·696	L.
			210·3	< 0·1	1	...	·746	L.
			210·6	< 0·1	1	...	·776	L.
			185·0	< 0 1	1	...	·860	L.
			196·5	< 0·1	1	...	·865	L.
			203·0	< 0·1	1	...	·959	L.
β 838	21 16	87 18	100·2	1·42	2	7·8 9·5	·742	B.
Σ 2799 ...	21 24	79 21	298·0	1·47	2	7·0 7·0	·842	W.B.
Σ 2804 ...	21 28	69 44	332·2	2·75	2	7·3 8·0	·853	W.B.
β 1212 ...	21 34	90 30	274·1	0·47	3	6·5 6·9	·738	B.
Σ 2822 ...	21 40	61 42	122·1	2·60	1	4·0 5·0	·868	L.
Ho. 166 ...	21 40	62 38	96·0	0·32	2	7·5 7·5	·690	L.
			90·6	0·34	1	...	·534	B.
β 989 A.B. (κ Pegasi)	21 40	64 49	243·2	0·11	1	3·9 5·0	·008	L.
			236·2	0·14	1	...	·036	L.
			210·3	0·11	1	...	·515	L.
			185·4	0·10	1	...	·542	L.
			194·2	0·12	1	...	·559	L.
			191·7	0·10	1	...	·636	L.
			167·4	0·06	1	...	·865	L.
			236·6	0·14	3	...	·030	W.B.
			182·2	0·09	1	...	·871	W.B.
			204·7	0·12	4	...	·680	B.
			194·7	0·10	2	...	·836	B.
Σ 2825 ...	21 42	89 36	116·2	0·91	4	8·0 8·2	·769	B.
Ho. 171 ...	21 48	62 41	173·6	0·84	2	8 8	·737	B.
			177·5	1·03	2	...	·850	L.
Ho. 179 ...	22 8	60 20	268·5	0·43	1	8·0 8·5	·835	W.B.
Σ 2878 ...	22 10	82 31	124·2	1·11	2	6·5 9·0	·814	B.
Σ 2881 ...	22 10	60 55	100·3	1·30	2	7·7 8·2	·786	B.
Ho. 180 ...	22 12	46 38	225·6	0·51	1	7·0 7·5	·953	B.
Σ 2900 ...	22 19	69 39	178·1	1·34	1	6·0 9·2	·835	W.B.
β 172	22 19	95 21	7·1	0·56	4	5·6 6·0	·745	B.
β 291	22 23	88 59	162·7	0·33	1	8·4 8·7	·830	B.
Σ 2912 ...	22 24	86 5	91·4	0·23	2	5·8 7·2	·814	B.
Σ 2934 ...	22 37	69 5	136·1	0·98	2	8·2 9·2	·786	W.B.

Star's Name.	R.A. 1900.	N.P.D. 1900.	Position Angle.	Distance.	No. of Nights.	Mags.		Epoch 1901.	Obs.
	h m	° ′	°	″					
Σ 2944 ...	22 43	94 44	260·1	3·41	2	7·0	7·5	·781	B.
OΣ 483 ...	22 54	78 48	222·3	0·94	2	6·2	7·7	·842	W.B.
β 79	23 12	92 4	87·1	1·03	3	8·0	8·6	·786	B.
β 80	23 14	85 8	19·6	0·23	3	8·0	8·8	·786	B.
β 1222 ...	23 23	86 59	38·0	1·20	2	9	9	·814	B.
β 1266 ...	23 25	59 44	14·3	0·22	4	8·9	9·0	·664	L.
β 720	23 28	59 14	342·3	0·32	3	5·9	6·0	·563	L.
			335·5	0·30	3	...		·859	B.
β 858	23 36	58 0	261·6	0·54	4	8·0	8·2	·669	L.
A. G. C. 14...	23 39	61 2	200·4	1·40	2	5·0	9·5	·941	L.
Barnard ...	23 42	85 19	170·0	0·60	1	8·6	8·6	·797	B.
Σ 3050 ...	23 54	56 50	214·4	2·55	2	6·0	6·0	·886	W.B.
β 733	23 57	63 26	239·6	0·66	3	6·0	11·0	·592	L.
Σ 3056 ...	23 59	56 17	142·6	0·57	1	7	8	·044	L.
			145·3	0·53	2	...		·792	B.
β 862	23 59	52 23	314·3	0·58	1	8·5	9·0	·044	L.

Capella.

Observer ... ... Mr. Lewis.

Date 1901. G.M.T.	Hour Angle.	Day from Beginning of the Year.	Position Angle.	Remarks.
d h			° ′	
Jan. 4 7¼	3½E.	3·3	36 8	Powers 670 and 1120. Observed without blue shade. Undoubtedly elongated, and quite as distinct as κ Pegasi—better definition here. Star surrounded by some fifteen rings. Measures of distance were made to verify the elongation. The diameter of the star was measured first in the direction of apparent elongation, and then at right angles to this direction, the measures being made by placing the inside edges of the wires tangential to the disc. (1) Sum of direct readings, 0ʳ·138. Sum of reverse readings, 0ʳ·115. ½ (difference), 0ʳ·0115. (2) Sum of direct readings, 0·135. Sum of reverse readings, 0·125. ½ (difference), 0ʳ·0050. If the discs are assumed equal, then the separation of centres is 0ʳ·0065 = 0″·076.

Date 1901. G.M.T.		Hour Angle.	Day from Beginning of the Year.	Position Angle.		Remarks.
d	h			°	′	
Jan 14	$7\frac{1}{2}$	4 E.	13·3	17	1	Powers 670 and 1120. Images very fine. A large number of rings, easier than κ Pegasi; but the definition was better when measuring Capella than it was when measuring κ Pegasi. Measures made in the same way as on January 4. (1) Sum of direct readings, $0^r{\cdot}174$. Sum of reverse readings, $0^r{\cdot}093$. $\frac{1}{2}$ (difference), 0^r 041. (2) Sum of direct readings, $0^r{\cdot}164$. Sum of reverse readings, $0^r{\cdot}094$. $\frac{1}{2}$ (difference), $0^r{\cdot}035$. If the discs are assumed equal, then the separation of centres is $0^r{\cdot}096 = 0''{\cdot}072$. κ Pegasi measured in the same way gave $0''{\cdot}108$. Both Capella and κ Pegasi were measured in the usual way by trying to make the centres of the wires tangential to the discs; the results were: $0''{\cdot}132$ for Capella. $0''{\cdot}137$ for κ Pegasi.
Mar. 26	$8\frac{1}{2}$	$3\frac{1}{2}$W.	84·3	285 or 105	7	Power 670. Elongation not so apparent as on January 14.
Apr. 1	$7\frac{3}{4}$	$3\frac{1}{2}$W.	90·3			Power 670. Definition very fine. A large number of rings; as nearly round as I have seen it. There may be an elongation in 213°·7 or 33°·7, which would probably be due to the shaking of the instrument. Windy.
9	$8\frac{3}{4}$	5 W.	98·3			Powers 670 and 1120. Definition fair. If any elongation it is in 272°·3 or 92°·3.
17	$7\frac{1}{2}$	4 W.	106·3			Powers 670 and 1120. Splendid definition. Star apparently round.
18	$7\frac{3}{4}$	$4\frac{1}{2}$W.	107·3			Powers 670 and 1120. Definition very good. A large number of rings. Actual disc very minute. A slight elongation suspected in 310°, which might be due to the trembling of the instrument.
23	$7\frac{1}{2}$	$4\frac{1}{4}$W.	112·3			Power 670. Definition very good. Image steady. Set at 20°, but could not detect any elongation.
29	$7\frac{1}{4}$	5 W.	118·3			Powers 670, 840, and 1120. Definition very good and image steady. With 670 and 840 the star appeared round; with 1120 a very slight elongation was suspected at 327°·5 or 147°·5.

Date 1901. G.M.T.		Hour Angle.	Day from Beginning of the Year.	Position Angle.		Remarks.
d	h			°	′	
Nov. 14	10½	3½E.	317·4			Powers 670 and 1120. Star nucleus very small indeed and beautifully defined. A large number of rings almost perfect and circular. If the disc is not round the elongation is at 263°·2 or 83°·2.
Observer		...	...	...	...	Mr. Bryant.
Mar. 1	3	3¾E.	59·1	207	25	Power 670. Clouds passing. Very windy. Occasional glimpses of fine definition through thin clouds. Two blue shades used.
Apr. 17	6	2½W.	106·3	30	48	Power 670. Definition fine through thin cloud. Elongation very slight.
20	0	3 E.	109·0	28	25	Power 670. Very windy. Sky very clear. Star very bright. Seeing at intervals excellent when wind dropped. Two blue shades used. With power 1120 effect of wind greater, preventing measures.
22	5½	2½W.	111·2	14	12	Power 670. Definition fair. Star small and steady. Thin clouds. Elongation very slight. Two blue shades used.
26	5½	2½W.	115·2	359	5	Power 670. Very windy. Definition fair momentarily. Clouds passing.
30	5¼	3 W.	119·2	344	55	Powers 670 and 1120. Definition very fine indeed, but elongation very slight. Measures give distance about 0″·08 (power 1120). Distance thought to be really less than measures indicate. Two blue shades used.
May 24	5½	4½W.	143·2	267	29	Power 670. Very windy, otherwise definition at times good. Blue shades used. Too windy to use higher power. Measures of distance give 0″·09.
July 4	0½	2½W.	184·0	135	25	Power 670. Hazy. Image very unsteady, but small. Two blue shades used.
11	0	2½W.	191·0	112	51	Power 670. Definition fair at long intervals. Sky hazy. Two blue shades used. Apparently this is the right quadrant.
Aug. 18	22	2¾W.	229·9	326	40	Power 670. Sky hazy. Definition fair. N.E. wind. Quadrant seems to be right. Used two shades or one or more as haze varied.
19	21¾	2½W.	230·9	322	42	Power 670. Through cloud. Two blue shades used. Star very bright and diffused. Strong N.E. wind.

Date 1901. G.M.T.		Hour Angle.	Day from Beginning of the Year.	Position Angle.		Remarks.
d	h			°	′	
Aug. 20	22¼	3¼W.	231·9	319	32	Powers 670 and 1120. Thin cloud. Two blue shades used. Definition not very good. Star diffused, generally with moments of better seeing. Measures of distance give 0″·078.
21	22½	3½W.	232·9	316	0	Powers 670 and 1120. Two blue shades used. Taken during lulls of wind. Definition at times very good.
Sept. 1	21½	3 W.	243·9	279	27	Powers 670 and 1120. Blue and green shades used. Through thin cloud. Definition varying. Measures of distance gave 0″·09.
Observer		...	...	...	...	Mr. Bowyer.
Jan. 2	7½	3¾E.	1·3	45	3	Powers 670 and 1120. Sky very hazy. Definition fair. Rings visible occasionally. Measures easier than December 29.
15	6¾	2¾E.	14·3	18	8	Powers 670 and 1120. Measures made with and without blue shade. Definition fair. Several rings visible occasionally. Image rather diffused with high power.
18	6¾	2½E.	17·3	11	0	Powers 670 and 1120. Definition fair, but image unsteady. Measures made with and without blue shade. Elongation seemed rather more apparent. Measures easier than for κ Pegasi.
Mar. 11	10¼	4½W.	69·4	176	2	Powers 670 and 1120. Measured through clouds when star appeared of about 8½ mag. Definition very fair at first, but as clouds became thicker definition got much worse.
Apr. 12	7½	3¾W.	101·3	53	9	Power 670. Measures made in twilight during short intervals of fair definition. Elongation very slight.

Σ 484 *and* Σ 485, *and Two Pairs.* By the Rev. T. E. Espin, M.A.

This fine group forms cluster H VII. 47. Mr. Holmes has given a drawing of it in the *English Mechanic*, 1901 December 27, and calls attention to the fact that there are several *comites* to Σ 485, and a pair similar to Σ 484 following. Two of the *comites*, D and E, had been previously seen and measured by Dembowski. Of the other *comites* F is fainter than either of Dembowski's, and the other two are more distant. When examining the group on January 17 with the 17¼-inch reflector a much nearer and more

difficult *comes* than any of these was suspected, and on January 28 and on subsequent nights it was seen and measured, but with great difficulty. The following are the previous results of the measures of this group, together with my own recent measures, which are, in general, the means of three nights' observations.

	Epoch.	Position Angle.	Distance.	Observer.
		°	″	
AB (Espin)	1902·09	257·0	5·43	Espin.
AC (Σ 485)	1830·24	303·32	17·98	Σ.
	1867·32	303·07	17·89	De.
	1902·09	303 30	17·45	Espin.
CD (Demb.)	1867·10	85·10	...	De.
	1902·10	84·75	14·15	Espin.
CE (Demb.)	1867·68	333·65	19·86	De.
	1902·09	333·07	18·87	Espin.
A^1B^1 (Σ 484)	1830·87	132·42	5·42	Σ.
	1867·32	130·80	5·38	De.
	1902·09	133·55	...	Espin.
A^1C^1	1830·87	244·05	48·96	Σ.
	1867·32	244·23	48·99	De.
	1902·11	244·7	48·8	Espin.
AF (Holmes)	1902·09	129·4	13·95	Espin.
A^2B^2 (Holmes)	1902·07	216·0	5·65	Espin.
CA^1	1902·11	244·7	48·8	Espin.
AA^2	1902·07	71·2	89·9	Espin.

I make the magnitudes as follows: A = 6·0; B = 12·0; C = 6·1; D = 10·8; D = 11·0; F = 11·5; A^2 = 9·5; B^2 = 10·5. Of Mr. Holmes's other two *comites* one is more distant than D in the same quadrant, and the other is about 10″ from E.

A comparison of the measures of the brighter stars shows that there is no motion within the group so far, and it would be interesting to know if a proper motion can be found for either of these stars.

In the course of the observations two pairs were found and measured :—

No. 1 B.D. +62°·640 R.A. $3^h\ 51^m\ 27^s$ Decl. +62° 10′·3 (1880).

			Mags.		
AB	P 221°·3	D 1″·48	8·2	8·5	1902·079
	220·3	1·48	8·2	8·6	·115
AC	169·7	8·2		10·5	·079
	169·4	8·6			·115

[A letter received since the above was written informs me that the pair No. 1 had been previously detected by Bird, and has been measured by Doolittle.]

No. 2 B.D. +61°·666 R.A. $3^h\ 55^m\ 19^s$, Decl. +61° 50′·5 (1880).

			Mags.	
AB	P 247°·5	D 4″·9	8·6 10·0	1902·079
	249·9	5·1	8·5 11·0	·112

The first is marked "Dupl ?" in the zones of the Astronomische Gesellschaft.

[*Note added by Mr. Lewis.*]

The two principal stars in this group are *Lalande* 7430 and 7433. The following are the places, brought up to 1890, from various catalogues :

		R.A. Lalande 7430. h m s	R.A. Lalande 7433. h m s
Lalande	1800	3 58 8·302	3 58 10·392
Oeltz. Arg. (N)	1842	8·301	10·101
Radcliffe	1845	8·176	10·576
Brussels	1865	8·122	10·322
Romberg	1878	8·227	10·297
Helsingfors	1881	8·227	10·377
Greenwich	1895	8·102	10·228

		N.P.D. Lalande 7430. ° ′ ″	N.P.D. Lalande 7433. ° ′ ″
Lalande	1800	27 57 52·1	27 58 4·1
Oeltz. Arg. (N)	1842	58·07	8·0
Radcliffe	1845	58·0	7·5
Brussels	1872	59·46	9·44
Romberg	1878	58·95	8·25
Helsingfors	1881	57·85	6·95
Greenwich	1890	59·52	9·45

These places are few and very precarious, and as the micrometer measures show no change it is fair to combine the two stars. The mean seconds for R.A. and N.P.D. 1890 are these :

	R.A. s	N.P.D. ″
Lalande	9·348	58·10
Oeltz. Arg.	·201	63·07
Radcliffe	·376	62·75
Brussels	·222	64·45
Romberg	·262	63·60
Helsingfors	·304	62·40
Greenwich	·165	64·49

results from which it is unsafe to deduce any proper motion. No systematic corrections have been applied.

Micrometrical and Visual Observations of Nova Cygni (1876) *made with the 40-inch Refractor of the Yerkes Observatory.* By E. E. Barnard.

On the evening of 1876 November 24, at $5^h\ 45^m$, Schmidt at Athens discovered a brilliant star of the 3rd magnitude in *Cygnus* where a few days previous none such had existed. It rapidly faded, and by December 15 had ceased to be visible to the naked eye. The period of visibility without the telescope, therefore, did not extend over more than twenty-two days.

The most rapid decline in its light occurred between 1876 November 27 and November 29, when, according to Schmidt, in less than two days it faded through nearly $1\frac{1}{2}$ magnitude. From then on until it ceased to be visible to the naked eye the decline was slow and gradual at about 0·13 magnitude a day. By 1877 February 1 it had reached 7·5 magnitude. By 1877 September 1 it had declined to the 10th magnitude, at which brightness it remained for nearly a year. Throughout 1880 and the first part of 1881 it remained constant at slightly less than 12 magnitude. On 1882 March 24 the Dun Echt observers estimated it to be of the 14th magnitude. There seem to be no other observations to verify this last estimation. Nearly ten years later (July 31, 1891) Professor Burnham examined the star with the 36-inch of the Lick Observatory and estimated it to be 13·5 magnitude. He found it a little brighter than the star No. 50, which he estimated to be 14 magnitude. I have not found any later observations of the star.

At the observations in 1891 (*Pub. L. O.*, vol. ii., p. 179), Professor Burnham says: "At times the new star did not seem to have a perfectly stellar appearance under moderately high powers, but rather to resemble an exceedingly minute nebula. This appearance, however, may not be real. The star is too faint to allow one to decide a question of this kind with any certainty. I did not make any measures from surrounding stars, as that has been very thoroughly done by the authors of the paper referred to [Copeland and Lohse: *Copernicus*, vol. ii., 101]."

Fearing the possible total disappearance of the *Nova*, Copeland and Lohse made an extensive series of measures of its position with respect to 112 surrounding stars. These observers state in *Copernicus*, vol. ii.: "Towards the end of 1877 October *Nova Cygni* had decreased still further in brightness, and so pointed to the possibility that it might eventually fade altogether from view. In the event of such a contingency it seemed very desirable to possess a map of the small stars in the immediate neighbourhood; accordingly measures in position-angle and distance from the new star to surrounding objects were commenced on October 29 and added to as occasion offered, until the whole series of measures extended to 38 nights, the last being 1882

March 24. The resulting map includes every star fairly measurable with the Dun Echt refractor of 15·06 inches aperture within $7\frac{1}{2}'$ of *Nova Cygni*. . . . In this place it may be well to particularise a very small star, probably about the 15th magnitude, at a distance of 19″·1, in position-angle 314°·2 from *Nova*, as in a superficial examination it might possibly be confounded with the new star in the event of the total disappearance of the latter."

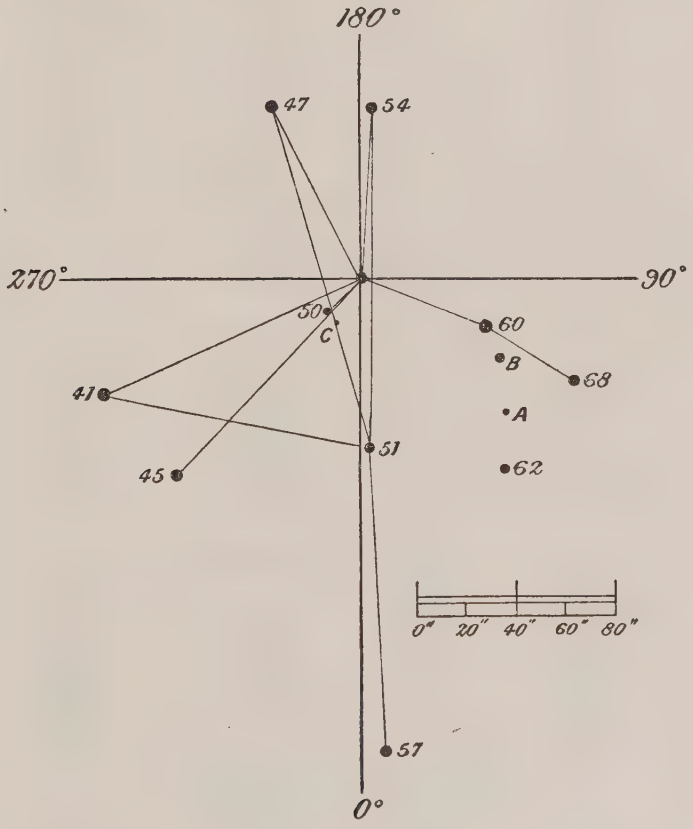
These measures and a most excellent map of 113 stars (including the *Nova*) are printed in *Copernicus*, Nos. 18 and 19, for 1882 June and July, for a copy of which I am greatly indebted to Dr. Ralph Copeland, Astronomer Royal for Scotland. The positions of these stars are reduced to the epoch 1878·0. I am also indebted to Dr. See for copies of the chart and of some of the measures in *Copernicus* from the library of the U.S. Naval Observatory.

To see if any change has occurred in the place of the new star in the past quarter of a century, I have selected a few of these stars for re-measurement along with the *Nova*. The re-measurement of all the stars observed at Dun Echt would be an unnecessary piece of labour, I therefore selected those stars near the *Nova* and which were within about two minutes of arc from it.

When I first looked up this object, on 1901 October 21, I adopted Schmidt's position of it given in *Ast. Nach.*, Bd. 89, p. 10, not having any chart by which to identify its place. Unfortunately Schmidt's position was in error (corrected later on in *A. N.*). Singularly enough the erroneous place coincided to within one second of time and a few seconds of arc, with a 15^m star (No. 51) which I observed that night and the next for the *Nova*. Upon deducing the place of this star from the observed $\Delta\alpha$ and $\Delta\delta$ and comparing it with the meridian positions obtained at Berlin (*A. N.* 2113, Bd. 89, p. 14), it was seen that the wrong star had been observed. This accounts for the measures of No. 51 on October 21 and 22. For the sake of accuracy I have since added one more night's measures to these stars. After this the *Nova* was easily identified.

I have charted the stars measured here with the 40-inch, and as a ready means of identifying the various measures at a glance I have not only added the numbers of the stars, but have also indicated the measures by lines connecting the various stars. Thus at a glance, without having resort to the actual measures, one can identify any star without trouble. It is to be regretted that diagrams of this kind are so often printed without any means of identifying any individual star without recourse to the measures and a protractor. Often I have been confused and uncertain in picking out the stars on such diagrams, without having a protractor immediately at hand to indicate the angles. In the chart in *Copernicus* of *Nova Cygni* this is not so bad, because the coordinates are printed on the map. But even here the identification of the individual stars would be much facilitated by numbering them on the chart. With such charts as I mention

it is an easy matter to identify the corresponding stars in the sky. But you still do not know the number of this or that particular star in the list of measures. If it happens to be at the telescope that a particular star on the chart is wanted to be identified by its number, as is frequently the case, it becomes very annoying and is the source of the loss of much time. I have, therefore, gone to the trouble of identifying the stars on such charts by scale and protractor, and numbering them to save time and confusion.



On the present chart the numbers are those assigned the stars at Dun Echt.

I have inserted four additional stars besides those measured. The star A is very faint = $16\frac{1}{2}$ magnitude. B is of the same brightness as 62, while C, the very faint star mentioned near 50,

is extremely difficult under not very fair conditions. The star 62 has been inserted because of the comparisons of the light of the *Nova* with it.

Following are the measures :—

Micrometer Measures of Stars near Nova Cygni (1876).

Nova and No. 60. Magnitude 11·4.

1901·844	Nov. 4	70°15	52″53
·846	5	70·48	53·10
·866	12	70·51	52·66*
·926	Dec. 4	70·58	52·47†
·961	17	70·25	53·06
·997	30	70·05	52·96
1901·907		70·34	52·80

These give $\Delta\alpha = 0^m\ 4^s{\cdot}488$, $\Delta\delta\ 0'\ 17''{\cdot}77$.

* Single distances; clouds.
† Two single distances; clouds.

Nova and No. 54. Magnitude 12·1.

1901·844	Nov. 4	175°27	67″83
·846	5	175·94	68·08
·926	Dec. 4	175·88	67·82
1901·872		175·70	67·91

Nova and No. 47. Magnitude 12·2.

1901·844	Nov. 4	206°27	76″67
·846	5	206·04	76·49
·826	Dec. 4	206·27	76·39
1901·839		206·19	76·52

Nova and No. 41. Magnitude 13·2.

1901·846	Nov. 5	294°01	110″09
·866	12	294·46	...*
·926	Dec. 4	294·01	110·02
·994	29	294·13	110·05
1901·908		294·15	110·05

* Clouds prevented distance measures.

Nova and No. 50. Magnitude 16·0.

1901·926	Dec. 4	312°·97	19″·38*
·961	17	315·54	20·09
·997	30	316·05	19·87
1901·961		314·86	19·78

* There is a 16ᵐ·8 star close north of 50.

Nova and No. 45. Magnitude 13·7.

1901·961	Dec. 17	316°·28	106″·17
·994	29	316·28	106·05
1902·005	Jan. 2	316·21	105·80
1901·987		316·26	106·01

Nova and No. 51. Magnitude 15·3.

1901·844	Nov. 4	2°·48	66″·60
·846	5	2·34	66·68
·882	18	2·32	66·64
·926	Dec. 4	2·11	66·71
1901·875		2·31	66·66

These give $\Delta\alpha = 0^m\ 0^s\cdot243$, $\Delta\delta = 1'\ 6''\cdot61$.

Positions of Stars with reference to No. 51.

No. 51 and No. 57. Magnitude of No. 57, 12·0.

1901·808	Oct. 22	2°·62	116″·83
1902·033	Jan. 12	2·57	117·07
·035	13	2·55	117·21
1901·959		2·58	117·04

No. 51 and No. 60.

1901·808	Oct. 22	135°·52	67″·98
1902·033	Jan. 12	135·35	68·19
1901·920		135·43	68·08

These give $\Delta\alpha = 0^m\ 4^s\cdot378$, $\Delta\delta = 0'\ 47''\cdot78$.

No. 51 and No. 54.

1901·808	Oct. 22	178°·92	133″·86
1902·033	Jan. 12	178·84	134·24
1901·920		178·88	134·05

No. 51 and No. 47.

1901·808	Oct. 22	195°08	140″18
1902·033	Jan. 12	194·64	140·17
1901·920		194·86	140·18

No. 51 and No. 41.

1901·808	Oct. 22	258°20	105″21
1902·033	Jan. 12	257·90	105·21
1901·920		258·05	105·21

No. 60 and No. 68. Magnitude of No. 68 = 13·5.

1902·071	Jan. 26	57°62	31″84
·074	27	57·77	31·73
1902·073		57·69	31·78

In general the magnifying power used was 460, but some of the measures were made with 700 diameters.

Following are comparisons of these measures with those made at Dun Echt in 1877–82 :—

Nova and No. 41.

Dun Echt	293°4	... ...	108″6
Yerkes	294·1	... ...	110·1
D. E. – Y.	= −0·7	... ...	−1·5

Nova and No. 45.

Dun Echt	315°0	... ...	107″9
Yerkes	316·3	... ...	106·0
D. E. – Y.	−1·3	... ...	+1·9

Nova and No. 47.

Dun Echt	206°2	... ...	78″2
Yerkes	206·2	... ...	76·5
D. E. – Y.	0·0	... ...	+1·7

Nova and No. 50.

Dun Echt	314°2	... ...	19″1
Yerkes	314·9	... ...	19·8
D. E. – Y.	−0·7	... ...	−0·7

Nova and No. 51.

Dun Echt	...	...	358°·5	...	...	63″·7
Yerkes ...	...	...	362·3	...	...	66·7
D. E.–Y.	...	...	−3·8	...	...	−3·0

Nova and No. 60.

Dun Echt	...	...	69°·7	...	...	55″·2
Yerkes ...	...	...	70·3	...	...	52·8
D. E.–Y.	...	...	−0·6	...	...	+2·4

No correction has been applied to the angles for precession.

These discrepancies are rather large. They do not seem to be due to any systematic errors nor to motion in the *Nova*. It is scarcely probable that they are due to motion in any of these small stars themselves.

The Dun Echt observers state, in *Copernicus* Nos. 18–19, p. 117, that "the probable error of one of the places is about ±1″·1 in either coordinate; this quantity is disappointingly large, but is probably to be ascribed in some measure to two special causes—the peculiar light of the *Nova*, which threw its image out of focus, and the fact that the settings could not all be made in the centre of the apparent field of view." The magnifying power used at Dun Echt was nearly always 229.

The star No. 60 is the one mentioned by Schmidt as a companion to the *Nova*, of the 12–13^m, 4^s·25 fol. and 15″·5 north. Schmidt's position was determined from two transits of a ring micrometer, and are not very exact (*A. N.* 2122, Bd. 89, p. 159).

Following are the observations for the position of the *Nova*, made with respect to the stars Bonn A. G. Cat. 15868 (D.M. +42°·4184) and 15876 (D.M. +42°·4185) :—

Observed Δα *and Measured* Δδ.

No. 51 and D.M. +42°·4184.

			m s		′ ″	
1901·805	Oct. 21	Δα	0 24·51	(10)	Δδ 2 20·41	(2)
·808	22		0 24·50	(14)	2 20·92	(3)
1901·806			24·50	(28)	2 20·71	(5)

No. 51 is north preceding.

No. 51 and D.M. +42°·4185.

			m s		′ ″	
1901·805	Oct. 21	Δα	0 34·96	(10)	Δδ 0 9·10	(2)
·808	22		0 34·90	(14)	0 9·67	(4)
1901·806			0 34·92	(28)	0 9·48	(6)

No. 51 is south preceding.

No. 60 and D.M. +42°·4185.

			m s		′ ″
1901·808	Oct. 22	Δα	...	Δδ	0 57·96 (2)
·827	29		0 30·65 (8)		0 58·01 (1)
1901·817			0 30·65 (8)		0 57·98 (3)

No. 60 is south preceding.

No. 60 and D.M. +42°·4184.

1901·827	Oct. 29	Δα	0m 20s·25 (8)	Δδ	1′ 32″·04 (2)

No. 60 is north preceding.

No. 51 and No. 60.

1901·805	Oct. 21	Δα	0′ 47″·50 (2)	Δδ	0′ 48″·42 (2)

$\frac{1}{15}$ sec δ(= 4s·29).

No. 51 is north preceding No. 60.

Nova and No. 60.

1901·827		Δα	0′ 51″·00 (4)	Δδ	0′ 17″·78 (5)

$\frac{1}{15}$ sec δ(= 4s·60) reject.

Nova is south preceding No. 60.

At the observations for Δα *Nova* was very difficult, and the measures should be rejected.

All the measures have been corrected for refraction.

As the comparisons with the Dun Echt measures do not seem to indicate with any certainty any motion in the *Nova*, it has occurred to me that this question might be settled by the comparisons of some of the stars and the *Nova* with the two D.M. stars following the new star, which have been observed on the meridian, and with the meridian observations of the *Nova* itself made some twenty-five years ago.

This investigation follows.

The star D.M. +42°·4184 has been observed at Bonn, Dun Echt, and Greenwich. These positions brought up to 1901·0 are in fair accord—

D.M. +42°·4184.

		h m s		° ′ ″	
1901·0	α	21 38 14·24	δ =	+42 22 10·54	Bonn Beob.
		21 38 14·41		42 22 11·08	Bonn A.G. Cat.
		21 38 14·41		42 22 10·48	Greenwich and Dun Echt.
1901·0	α =	21 38 14·38	δ =	+42 22 10·65	

Equal weight is given in these to the different observatories.

The star D.M. +42°·4185 has been observed at Bonn.

1901·0 $21^h\ 38^m\ 24^s{\cdot}60$ $+42°\ 24'\ 39''{\cdot}5$ Bonn A.G. Cat

From the first of these stars (42°·4184) we get the following position for star 51 from the observed $\Delta\alpha$ and $\Delta\delta$.

Star 51 1901·0 $\alpha = 21^h\ 37^m\ 49^s{\cdot}88$ $\delta = +42°\ 24'\ 31''{\cdot}36$ (A)

And this, with the observed position-angle and distance, gives the following position :

Nova 1901·0 $\alpha = 21^h\ 37^m\ 49^s{\cdot}67$ $\delta = +42°\ 23'\ 24''{\cdot}70$. (1)

From the same star (42°·4184) the following position of No. 60 is obtained from the observed differences of α and δ :

Star 60 1901·0 $\alpha = 21^h\ 37^m\ 54^s{\cdot}13$ $\delta = +42°\ 23'\ 42''{\cdot}69$ (*a*)

From the position-angle and distance this gives for the

Nova 1901·0 $\alpha = 21^h\ 37^m\ 49^s{\cdot}64$ $\delta = +42°\ 23'\ 24''{\cdot}92$ (2)

From the second star (42°·4185) we similarly deduce the following position of

No. 51 1901·0 $\alpha = 21^h\ 37^m\ 49^s{\cdot}67$ $\delta = +42°\ 24'\ 30''{\cdot}12$ (B)

From which the position of the *Nova* is derived as before.

Nova 1901·0 $\alpha = 21^h\ 37^m\ 49^s{\cdot}43$ $\delta = +42°\ 23'\ 23''{\cdot}51$ (3)

This star (42°·4185) also gives the place of

No. 60 1901·0 $\alpha = 21^h\ 37^m\ 53^s{\cdot}95$ $\delta = +42°\ 23'\ 41''{\cdot}52$ (*b*)

And this in turn gives for the *Nova* :

Nova 1901·0 $\alpha = 21^h\ 37^m\ 49^s{\cdot}46$ $\delta = +42°\ 23'\ 23''{\cdot}75$ (4)

Collecting these four positions of the *Nova* for 1901·0 we have

	h m s	° ′ ″
(1)	$\alpha = 21\ 37\ 49{\cdot}67$	$\delta = +42\ 23\ 24{\cdot}70$
(2)	21 37 49·64	42 23 24·92
(3)	21 37 49·43	42 23 23·51
(4)	21 37 49·46	42 23 23·75
1901·0	$\alpha = 21\ 37\ 49{\cdot}55$	$\delta = +42\ 23\ 24{\cdot}22$

The two first positions, depending on star 42°·4184, and the two last, on star 42°·4185, show a systematic difference amounting to $0^s{\cdot}2$ and $1''$ in α and δ respectively. This is perhaps due to motion in one or the other of the two stars.

Collecting the various meridian observations of the *Nova*, made when it was in its brighter stages, I find there were four observations. Bringing these up to 1901·0 we have

Positions of the Nova from Meridian Observations.

	h m s	° ′ ″ W.	
1901·0	α = 21 37 49·41	δ = +42 23 24·20 (2)	Vienna.
	21 37 49·47	42 23 23·98 (4)	Berlin.
	21 37 49·55	42 23 23·33 (5)	Greenwich.
	21 37 49·40	42 23 25·33 (1)	Dun Echt.
1901·0	α = 21 37 49·49	δ = +42 23 23·86	

The weights assigned there are from the number of observations. One of the observations at Vienna was given half weight, as it was observed under poor conditions.

Comparison of the mean of these positions with that of mine shows no certain change in the place of the *Nova*.

The agreement would be closer if we used the observations referred to 42°·4185 alone. As this last star place rests on nine observations it would seem that there has been a small change in the place of 42°·4184 and no change in that of the *Nova*.

The magnitudes I have assigned to the stars are the mean in each case of several estimates.

Following are comparisons of the magnitudes :—

Star 41.	m
Dun Echt	13·4
Yerkes	13·2
D. E.–Y.	+0·2

Star 49.	m
Dun Echt	14·7
Yerkes	14·0
D. E.–Y.	+0·7

Star 42.	
Dun Echt	15·0
Yerkes	14·0
D. E.–Y.	+1·0

Star 50.	
Dun Echt	15·6
Yerkes	16·0
D. E.–Y.	−1·0

Star 45.	
Dun Echt	14·4
Yerkes	13·7
D. E.–Y.	+0·7

Star 51.	
Dun Echt	14·3
Yerkes	15·3
D. E.–Y.	−1 0

Star 4	
Dun Echt	11·9
Yerkes	12·2
D. E.–Y.	−0·3

Star 54.	
Dun Echt	12·0
Yerkes	12·1
D. E.–Y.	−0·1

Star 57.		Star 68.	
Dun Echt	13·4	Dun Echt	14·0
Yerkes	12·0	Yerkes	13·2
D. E.−Y.	+1·4	D. E.−Y.	+0·8

Star 60.		Star 73.	
Dun Echt	11·0	Dun Echt	13·4
Yerkes	11·4	Yerkes	12·5
D. E.−Y.	−0·4	D. E.−Y.	+0·9

Collecting these differences of magnitude, we have :—

m	m
+0·2	−0·1
+1·0	+1·4
+0·7	−0·4
−0·3	+0·8
+0·7	+0·9
−1·0	D. E.−Y. = +0·2
−1·0	

The magnitudes of those stars which were not micrometrically measured by me should not have much weight, as they would not be so carefully estimated.

These discordances of magnitude are not very large, and no more than would be expected from different observers with different telescopes and such faint objects as some of these stars are.

It must be understood, however, that these estimates have been made under the most unfavourable conditions of seeing here, when a star anyways near the limit of the 40-inch would not be seen.

On several occasions I have seen a very faint star near No. 50 and north of it. It was too faint to measure—and there was no reason for measuring it.

By estimation, in terms of other distances and position-angles, this small star would be, with reference to No. 50, in

P. 20°± Dist. 5″±.

No. 50 is a faint star to have been measured at Dun Echt. There does not seem to have been any change in its brightness, though one would almost infer that it must have been brighter some twenty or twenty-five years ago.

It is difficult to estimate the brightness of the *Nova*. It seems to be more affected by bad seeing or moonlight than the small stars near it. This is another indication of its probable nebulous

character. My estimates of its relative light seemed to show that it varied through about $\frac{1}{2}$ magnitude. This, I think, however, is due to its lack of exact definition and the effect of local conditions of moonlight or bad seeing, though I am not sure yet that there is not a fluctuation of its light.

Following are estimations of the brightness of the *Nova* with respect to the stars 50 and 51.

Comparisons of the Brightness of the Nova.

1901 Nov.	12	*Nova*	= 51 and possibly slightly brighter.					
	18	,,	= 51.					
	19	,,	= $\frac{5}{10}$ m less than	51.				
Dec.	4	,,	= $\frac{2}{10}$ m	,,	,,	$\frac{2}{10}$ m	brighter than	50.
	17	,,	= $\frac{3}{10}$ m	,,	,,	$\frac{5}{10}$ m	,,	,,
	22	,,	= $\frac{5}{10}$ m	,,	,,	$\frac{1}{10}$ m	,,	,,
	29	,,	= 51	...	...	$\frac{5}{10}$ m	,,	,,

The following estimates are more carefully made and are believed to be more exact. They were made specially to decide the question of variability.

1901 Dec.	30	*Nova*	$\frac{1}{10}$ m	less than	51	...	$\frac{4}{10}$ m	brighter than 50.
1902 Jan.	2		midway between 50 and 51.					
	5		$\frac{3}{10}$ m	less than	51	...	$\frac{1}{10}$ m	,, ,,
	6		$\frac{3}{10}$ m	,,	,,	...	$\frac{5}{10}$ m	,, ,,
	7		$\frac{2}{10}$ m	,,	,,	...	$\frac{5}{10}$ m	,, ,,
	10		$\frac{2}{10}$ m	,,	,,	...	$\frac{4}{10}$ m	,, ,,
	11		$\frac{3}{10}$ m	,,	,,	..	$\frac{3}{10}$ m	,, ,,
	12		$\frac{3}{10}$ m	,,	,,	...	$\frac{3}{10}$ m	,, ,,
	13		$\frac{3}{10}$ m	,,	,,	...	$\frac{2}{10}$ m	,, ,,
	14		$\frac{2}{10}$ m	,,	,,	...	$\frac{4}{10}$ m	,, ,,
	18		$\frac{2}{10}$ m	,,	,,	...	$\frac{4}{10}$ m	,, ,,
	26		$\frac{2}{10}$ m	,,	,,	...	$\frac{4}{10}$ m	,, ,,
	27		$\frac{4}{10}$ m	,,	,,	...	$\frac{2}{10}$ m	,, ,,
Means			0·25				0·34	

These estimates do not seem to be decisive. It is rather probable though that, on account of the physical appearance of the *Nova*, these differences do not indicate variability. They may prove valuable, however, in the future in deciding any question of change.

With these, and the estimated magnitudes of stars 50 and 51,

the following two determinations of the present brightness of *Nova Cygni* result from star 50, $15^{m}\cdot7$, and from star 51, $15^{m}\cdot6$.

From ten nights' comparisons the star No. 62 was found to be equal to No. 50 (the mean gave $0^{m}\cdot03$ less than No. 50), while from eleven nights' estimates, No. 62 was found to be $0^{m}\cdot35$ less than the *Nova*.

Twelve nights' comparisons made No. 51 less than No. 68 by $0^{m}\cdot18$.

In all the estimates of brightness the eyes were placed parallel to the line joining the stars. This is very important in my case, for, as I have shown in *A. J.* 370, vol. xvi. p. 75, two stars that are equal when the line between them is parallel to my eyes, will differ by half a magnitude when they are in a vertical line; the lower one being always the brighter. Just what is the cause of this peculiarity is not clear, but it seems that it might be due to a dulling of a portion of the retina by the preponderance of the illumination of the sky in the daytime over that of the landscape, about one half of the retina usually receiving the light from the sky, while the other half receives that from the landscape. The lower star would therefore fall on that part of the retina usually occupied by the landscape, and hence on a more sensitive portion.

Mr. J. A. Parkhurst has kindly measured the relative light of Nos. 50 and 51 with the photometer on the 40-inch. The difference as determined by him is $0^{m}\cdot66$ from one night's observations under rather unfavourable conditions. This is a little larger than the difference that would be derived from my estimates with the *Nova* and these two stars, and agrees almost exactly with the difference of my estimates of the magnitudes ($0^{m}\cdot7$) of the two stars themselves.

The changes of colour in this star were not very remarkable. At the time of discovery by Schmidt it was of a strong golden yellow. This colour, according to Schmidt, it retained until it ceased to be visible to the naked eye—to him it was at no time of a decided orange tint, but yet of a full yellow. When observable only with the telescope some of the observers described it as red.

According to Dr. Copeland, at Dun Echt, the star presented a rather complicated spectrum during its brighter stages in 1876 and the beginning of 1877. On 1876 December 5, when its magnitude was 4·5, he found that one of the spectral lines coincided almost exactly with the brightest line of the gaseous nebulæ. Another line corresponded very nearly with one of the bright lines in the spectra of the Wolf-Rayet stars in *Cygnus*. In the next eight or nine months the spectrum was reduced to a single line with the merest traces of a continuous spectrum. The star itself was later described as having "a small disc with a soft margin;" the *Nova* seemingly having become a nebula, both visually and spectroscopically. In reference to this remarkable change in the physical condition of the star, Dr. Copeland says, in *A. N.* 2158:—"Bearing in mind the history of this star from

the time of its discovery by Schmidt, it would seem certain that we have an instance before us in which a star has changed into a planetary nebula of small angular diameter. At least it may be safely affirmed that no astronomer discovering the object in its present state would, after viewing it through a prism, hesitate to pronounce as to its present nebulous character."

One striking feature about *Nova Cygni* mentioned by the Dun Echt observers was the fact that there was a decided difference in the focal adjustment for it and a star. According to Dr. Copeland, with the 15-inch Grubb refractor, this amounted on 1877 October 2, when the star was about 10^m, to about 2 mm. (the focus being outside of that for a star). In this respect it resembled *Nova Aurigæ* rather than *Nova Persei* of the past year. There seems to be no record of anything of this kind in its earlier stages of brightness in 1876, and it probably showed no such peculiarity then.

So far as I can learn, *Nova Aurigæ* in its first appearance, when bright, did not show this peculiarity, nor even when faint in the spring of 1892. It was only at its second apparition that the effect became evident. At present *Nova Persei* has shown nothing of this kind. In all probability, however, it will show it later on when it has become much less bright. It is the intention to frequently test the focus to see when this effect first becomes visible—if it makes its appearance at all. Careful tests up to 1902 January 31 showed no sensible difference between its focus and that of a star.

On account of the faintness of *Nova Cygni* it is difficult now to measure its focus with reference to a star in the ordinary way. Under the best seeing, however, there is certainly a slight difference of focus to the extent of possibly $\frac{1}{10}$-inch outside of that for a star. Even when in best focus it does not appear to be perfectly sharp. It seems to be more or less ill-defined, resembling in this the present appearance of *Nova Aurigæ*. Of course no colour is ever discernible in so faint an object.

I have examined *Nova Coronæ* (1866) several times lately, but always under very bad conditions of seeing. Under those conditions no difference of focus was noticeable between it and a star—though I hope to investigate this point more carefully later on with better seeing.

I have mentioned the want of *Nova Persei* to show any peculiarity in its focus from that of a fixed star. There is another peculiarity which I believe is important, but which in the greater interest due to the photographic nebulosity about the *Nova* is lost sight of. In the *Astrophysical Journal* for 1901 October, vol. xiv. No. 3, attention is called to the peculiar appearance of the *Nova* in the 40-inch: "On August 12, the seeing was unusually good; examining the *Nova* with high powers I was struck with its appearance. It was brighter than the star [D.M. $+43^\circ.732$], but under high magnifying powers its light was strikingly dull, having more the appearance of planetary

light. This was so decided, that had I been examining the stars in that region I should at once have singled out the *Nova* as different from any of the stars. The spurious disc appeared much duller and somewhat larger than that of a star, and of a slight yellowish colour."

As the new star has faded, this peculiarity has become even more striking. The light of the *Nova*, which is greenish white, is very much like that of *Neptune*, and when *Neptune* is seen with a power that just begins to show its disc, the general likeness to the *Nova* would be striking. The light, however, is more greenish than that of *Neptune*. It also strongly resembles *Ceres* when that planet is seen with not too high a power. The colour is also like some of the planetary nebulæ—especially N.G.C. 7662 in *Andromeda*. While the light from a star is vivid, that of *Nova Persei* is singularly dull with high powers.

Yerkes Observatory, Williams Bay, Wisconsin:
1902 *February* 5.

On the Variation of S Carinæ. By Alex. W. Roberts, D.Sc.

There are at present (February 1902) between 160 and 180 stars south of $-30°$ dec., whose variation is known with some degree of certainty.

Of this number ten are of marked interest, as their variation was discovered and definite minima and maxima determined by Dr. Gould. The star *S Carinæ* (Ch. 3637) is one of these Cordoba variables. Inasmuch as I have already dealt with the variation of *R Carinæ* (*Monthly Notices*, vol. lxi. p. 552) and with the variation of *R Centauri* (*Monthly Notices*, vol. lxi. p. 355), both stars whose variation was discovered and defined by Dr. Gould, it will, I think, give a measure of completeness to these papers to consider also the variation of *S Carinæ*.

It may be noted here that the three stars are good examples of three well-defined types of long-period variation.

We consider in the first place the data which are of service as leading to a determination of the period of the star.

Dr. Gould's note on the variation of *S Carinæ* is as follows (*Uranometria Argentina*, p. 253):—

"The fluctuations of the light of this star appear to be within the limits $6\frac{1}{4}^m$ and a little above 9^m, and the length of its period to be several months. In 1872 its magnitude was observed as follows: T., May 26,* 7·0; April 3, 6·9; H., May 24, 30, 8·0; R., July 28, 7·9; Aug. 7, 7·4. In 1874 it was twice observed by Mr. Thome, May 21, 6·3; June 9, 7·3; observations which place the variability of the star beyond question.

* March 26?

"In May 1877 he estimated it as $8\frac{3}{4}^m$. Amid the great amount of other labour, sufficient observations have not been made to enable me to determine the light curve; but with the certainty of its variability I have not hesitated to designate it *S Carinæ*. Its colour is decidedly reddish."

Although Gould's observations are not sufficient to yield a period they are so situated on the light curve that they afford an indication of a definite date when the star passed through its maximum phase in 1872.

Later on in the present investigation this date will be determined and a period deduced from it. It is necessary first, however, to consider the more recent observations. These alone are sufficient to yield complete elements of variation, and thus they readily admit of being related to the Cordoba observations.

Observations of *S Carinæ* were begun at Lovedale in April 1891, and have been continued with one short interval down to the present. The number of observations secured during this period is close on 800. Nearly all the observations were made with a good 1-inch telescope, the usual precautions being taken to eliminate position error.

In the following two tables are given the several maxima and minima determined from the Lovedale observations. For purposes of computation the Julian day is given in the third column. In the fourth column is set forth the magnitude at maximum or minimum phase. In the last column is given the difference between these observed magnitudes and the mean magnitude for the whole series, viz. $6^m{\cdot}2$ for maximum and $8^m{\cdot}9$ for minimum.

TABLE I.

Determination of Maxima.

Rotation No.	Date.	Julian Day. J	Mag. m	Diff. m
1	1891 June 30	2411914	6·2	0·0
2	1892 April 17	2412206	5·8	−0·4
3	1893 Feb. 7	2412502	5·9	−0·3
4	June 30	2412645	5·9	−0·3
5	Nov. 30	2412798	6·0	−0·2
6	1894 May 9	2412958	6·6	+0·4
7	Sept. 27	2413099	6·0	−0·2
8	1895 July 21	2413396	5·7	−0·5
9	Dec. 28	2413556	6·7	+0·5
10	1896 May 21	2413701	6·0	−0·2
11	Oct. 16	2413849	5·8	−0·4
12	1898 Oct. 24	2414587	6·3	+0·1
13	1899 Mar. 28	2414742	6·6	+0·4
14	Aug. 20	2414887	6·3	+0·1

Rotation No.	Date.	Julian Day. J	Mag. m	Diff. m
15	1900 Jan. 24	2415044	6·6	+0·4
16	June 11	2415182	6·0	−0·2
17	Nov. 10	2415334	6·2	0·0
18	1901 April 7	2415482	6·4	+0·2
19	Aug. 29	2415626	5·8	−0·4
20	1902 Jan. 22	2415772	6·6	+0·4

TABLE II.

Determination of Minima.

Rotation No.	Date.	Julian Day. J	Mag. m	Diff. m
1	1892 Feb. 8	2412137	9·4	+0·5
2	July 6	2412286	9·2	+0·3
3	Nov. 19	2412422	8·8	−0·1
4	1893 May 3	2412587	9·0	+0·1
5	Sept. 20	2412727	8·7	−0·2
6	1894 Feb. 15	2412875	9·2	+0·3
7	July 15	2413025	9·3	+0·4
8	1895 May 13	2413327	8·7	−0·2
9	1896 Mar. 13	2413632	8·7	−0·2
10	Aug. 2	2413774	8·9	0·0
11	1898 Aug. 18	2414520	9·0	+0·1
12	1899 Jan. 9	2414664	8·9	0·0
13	June 8	2414814	8·9	0·0
14	Oct. 29	2414957	8·9	0·0
15	1900 Mar. 27	2415106	8·7	−0·2
16	Aug. 25	2415257	8·7	−0·2
17	1901 Jan. 24	2415409	8·7	−0·2
18	June 15	2415551	9·1	+0·2
19	Nov. 19	2415708	9·3	+0·4

It is not necessary to give prominence to the minor details of this investigation, which has for its purpose the determination of the elements of *S Carinæ* from the foregoing data. Indeed it would be tedious to do so. I may therefore confine myself to a statement of the main points and results of the investigation.

(1) If the dates of maximum or minimum phase be grouped in sets of three years, and if a mean period be obtained from each of these sets, there will appear evidence that the period of the star is subject to secular variation

It would seem that the cycle is completed in about nineteen years, or forty-five light periods, although on this matter the evidence is far from conclusive, as it must of necessity be, from

the limited period during which systematic observations have been made.

(2) Let now the following values be considered :—

$P + \Delta p$ = period of variation.

$T_M + \Delta t_M$ = epoch of maximum phase.

$T_m + \Delta t_m$ = epoch of minimum phase.

E = number of light periods dating from epoch.

J_M = Julian day of observed maximum (Table I. col. 3).

J_m = Julian day of observed minimum (Table II. col. 3).

We may regard the secular variation of period to be of the character represented by the expression

$$\kappa \cos (\phi E - M).$$

As a first approximation we take,

$$\phi = 8^\circ ;$$

that is, the cycle of perturbations is completed in forty-five light periods.

It is evident that, with regard to maxima determinations (Table I.), the relation between the foregoing quantities is set forth by the equation :—

$$(T_M + \Delta t_M) + E(P + \Delta p) + \kappa \cos (8^\circ E - M) = J_M \quad . \quad . \quad (1)$$

Let now $x = \kappa \cos M$

$= \kappa \sin M,$

and equation (1) becomes for maxima determinations

$$\Delta t_M + E\Delta p + x \cos 8^\circ E + y \sin 8^\circ E = \{J_M - (T_M + EP)\} \quad (2)$$

and for minima determinations

$$\Delta t_m + E\Delta p + x \cos 8^\circ E + y \sin 8^\circ E = \{J_m - (T_m + EP)\} \quad (3)$$

Thirty-nine equations of condition, twenty for Table I. and nineteen for Table II., were constructed on this type, the following values being assumed :—

$$P = 149^d$$

$$T_M = 2415038^J$$

$$T_m = 2415112^J.$$

(3) It will suffice to state only the final results of the solution of the two sets of equations.

Maxima determinations.—A rigorous consideration of the twenty equations yielded the following values :—

$$\Delta p = -0^{d}\cdot29$$
$$\Delta t_{M} = -5'57$$
$$x = +3\cdot74$$
$$y = -3\cdot56$$

that is,

$$P + \Delta p = 148^{d}\cdot71$$

$$T_{M} + \Delta t_{M} = 2415032^{J}\cdot4$$

Secular variation

$$= 5^{d}\cdot2 \cos (8° E - 316°\cdot3).$$

Minima determinations.—A similar but quite distinct solution of the nineteen equations (Table II.) yielded the following values :—

$$\Delta p = -0^{d}\cdot28$$
$$\Delta t_{m} = -4\cdot10$$
$$x = +1\cdot91$$
$$y = -3\cdot56$$

that is

$$P + \Delta p = 148^{d}\cdot72$$

$$T_{m} + \Delta t_{m} = 2415107^{J}\cdot9$$

Secular variation

$$= 4^{d}\cdot1 \cos (8° E - 331°\cdot7).$$

The close agreement between the results obtained from a separate consideration of the data in Table I. and the data in Table II. is strong presumptive evidence of the accuracy, not only of the value of the period obtained, but more especially of the amount and character of the secular variation.

Combining the two sets of results we have as the final and definitive values of the elements of *S Carinæ* :—

$$J_{M} = 2415032^{J}\cdot4 + E\,148^{d}\cdot72 + 4^{d}\cdot7 \cos (8° E - 324°)$$
$$M - m = 73^{d}\cdot2.$$

(4) In Tables III. and IV. is exhibited the amount of accordance between theory and observation.

The third column in both tables contains the observed dates

already given in Tables I. and II.; and the fourth column the dates computed from the elements just determined. The residuals are indicated in the last column.

Table III.

Maxima Observations.

Rotation No.	E.	Observed Date. J	Computed Date. J	O−C. d
1	−21	2411914·0	2411906·2	+7·8
2	19	2412206·0	2412204·6	+1·4
3	17	2412502·0	2412503·4	−1·4
4	16	2412645·0	2412652·8	−7·8
5	15	2412798·0	2412802·1	−4·1
6	14	2412958·0	2412951·4	+6·6
7	13	2413099·0	2413100·7	−1·7
8	11	2413396·0	2413399·4	−3·4
9	10	2413556·0	2413548·6	+7·4
10	9	2413701·0	2413697·7	+3·3
11	8	2413849·0	2413846·7	+2·3
12	3	2414587·0	2414590·8	−3·8
13	2	2414742·0	2414739·4	+2·6
14	−1	2414887·0	2414887·8	−0·8
15	0	2415044·0	2415036·2	+7·8
16	+1	2415182·0	2415184·5	−2·5
17	2	2415334·0	2415332·7	+1·3
18	3	2415482·0	2415480·9	+1·1
19	4	2415626·0	2415629·0	−3·0
20	+5	2415772·0	2415777·1	−5·1

Mean residual $3^{d}\cdot8$

Table IV.

Minima Observations.

Rotation No.	E.	Observed Date. J	Computed Date. J	O−C. d
1	−20	2412137·0	2412130·9	+6·1
2	19	2412286·0	2412280·1	+5·9
3	18	2412422·0	2412429·4	−7·4
4	17	2412587·0	2412578·9	+8·1
5	16	2412727·0	2412728·3	−1·3

Rotation No.	E.	Observed Date. J	Computed Date. J	O—C. d
6	−15	2412875·0	2412877·6	−2·6
7	14	2413025·0	2413026·9	−1·9
8	12	2413327·0	2413325·6	+1·4
9	10	2413632·0	2413624·1	+7·9
10	9	2413774·0	2413773·2	+0·8
11	4	2414520·0	2414517·7	+2·3
12	3	2414664·0	2414666·3	−2·3
13	2	2414814·0	2414814·9	−0·9
14	− 1	2414957·0	2414963·3	−6·3
15	0	2415106·0	2415111·7	−5·7
16	+ 1	2415257·0	2415260·0	−3·0
17	2	2415409·0	2415408·2	+0·8
18	3	2415551·0	2415556·4	−5·4
19	+ 4	2415708·0	2415704·5	+3·5

Mean residual $3^d·7$.

(5) In Tables I. and II. is given the libration on either side of the mean magnitude at maximum and minimum phase.

There was some expectation that this libration would, after the manner of the inequalities in period, follow some regular law.

It is evident, however, that the residuals given in the last column of Tables I. and II. are not systematic in their character; or, at any rate, there is so little evidence of systematic discordance that we may safely assume that it does not exist.

(6) It is pertinent to this investigation to relate the Cordoba observations of *S Carinæ* to those made at Lovedale.

Dr. Gould's observations are as follows :—

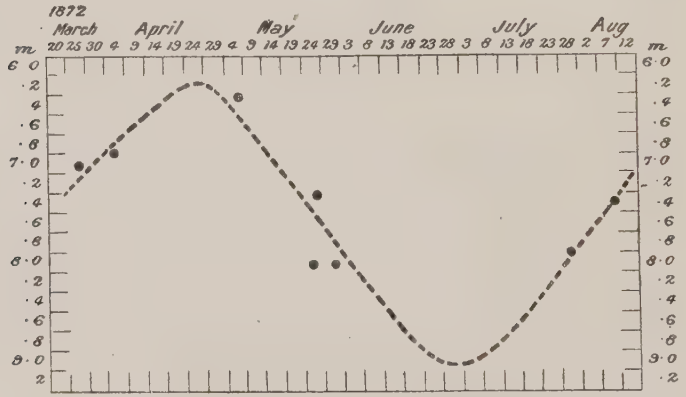
Observed Date.	Reduced Date.	Mag.
1872 Mar. 26	1872 Mar. 26	7·0
April 3	April 3	6 9
May 24	May 24	8·0
,, 30	,, 30	8·0
July 28	July 28	7·9
Aug. 7	Aug. 7	7·4
1874 May 21	May 6	6·3
June 9	,, 25	7·3

In the second column of the above table are given the dates of the first column reduced to one mean light period, that of March to August 1872.

Placed in order of time sequence, these observations are :—

	1872.	Mag.
1	Mar. 26	7·0
2	April 3	6·9
3	May 6	6·3
4	,, 24	8·0
5	,, 25	7·3
6	,, 30	8·0
7	July 28	7·9
8	Aug. 7	7·4

It is evident from the following chart that these eight observations indicate a maximum of $6^{m}\cdot2$ on or about 1872 April 24, and a mimimum of $9^{m}\cdot0$ on 1872 July 1.



Thus we have for an auxiliary determination of period :—

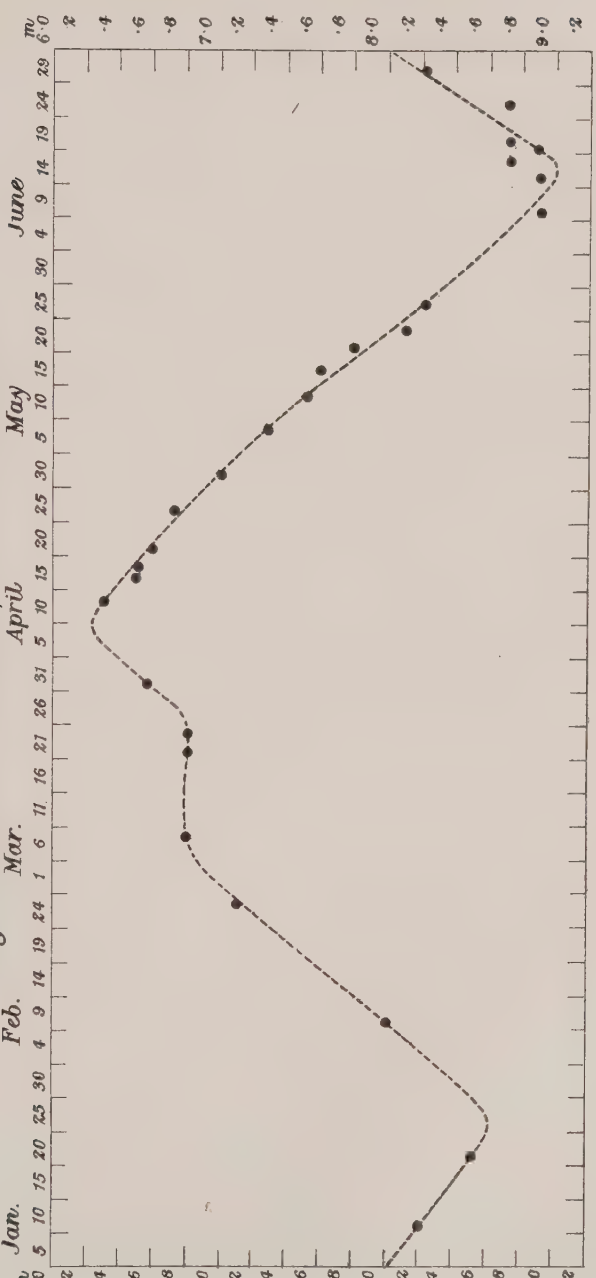
Maximum Phase.

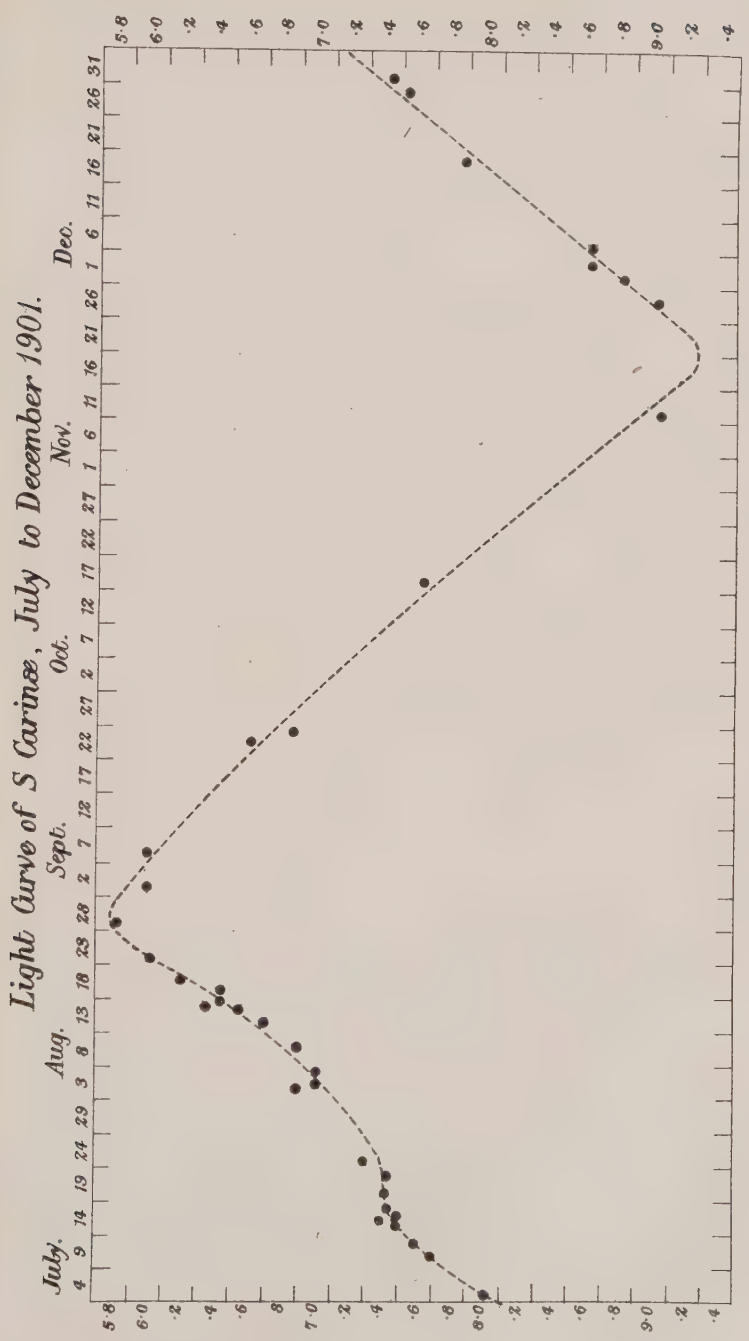
		d.
Cordoba observations,	1872 April 24	= 2404910·0
Lovedale observations,	1900 Jan. 12·4	= 2415032·4
	Interval =	10122·4

Minimum Phase.

		d.
Cordoba observations,	1872 July 1	= 2404976·0
Lovedale observations,	1900 March 28	= 2415107·9
	Interval =	10131·9

If we take the mean interval between the Cordoba and the


Light Curve of S Carinæ, Jan to June 1901.
Jan.
Feb.
Mar.
April
May
June
m
6·0
7·0
8·0
9·0


Light Curve of S Carinæ, July to December 1901.
July. Aug. Sept. Oct. Nov. Dec.
4 9 14 19 24 29 3 8 13 18 23 28 2 7 12 17 22 27 2 7 12 17 22 27 1 6 11 16 21 26 1 6 11 16 21 26 31
5·8 6·0 ·2 ·4 ·6 ·8 7·0 ·2 ·4 ·6 ·8 8·0 ·2 ·4 ·6 ·8 9·0 ·2 ·4

Lovedale mean light curves as 10127 days, we find that sixty-eight periods are performed in this time, giving a mean period of

$$148^{d}\cdot9.$$

This value is in satisfactory agreement with that found from Lovedale observations alone, viz.

$$148^{d}\cdot72.$$

If we take a mean between these two values we shall arrive at a value of the period correct to within one-tenth of a day.

(7) In plates 9 and 10 are represented graphically the light changes of *S Carinæ* during the year 1901.

The main characteristics of the light curve are (1) the practically equal duration of the increasing and decreasing phases, and (2) the indistinct secondary maximum which takes place about forty days before primary maximum.

With regard to the equality that exists between decreasing and increasing phases, the numerical values are :—

	d.
Increasing phase	73·2
Decreasing phase	75·5
Difference	2·3

It has been shown that a similar tendency to equality between increasing and decreasing phase exists in the case of the long period *T Centauri* (*Monthly Notices*, vol. lxii. p. 69).

In this case the values are :—

	d.
Increasing phase	44·1
Decreasing phase	46·5
Difference	2·4

In neither of these instances, therefore, is a rapid ascent to maximum a feature of the star's variation. Similar instances could be multiplied, but these two definite determinations are sufficient to form an exception of some weight to the view which sometimes prevails that in all cases of long period variation the rise to maximum is very much more rapid than the fall to minimum.

The secondary maximum, which is of a very rudimentary character, is sometimes entirely absent. In one of the 1901 light curves there is only slight evidence of its existence. The phenomenon is therefore irregular in its nature.

Apart from this exception, however, all the other features of variation of *S Carinæ* are of marked regularity. Its return to maximum or minimum can be predicted within a few days ; its limits of variation are subject to no erratic fluctuations, and there

is a certain general conformity to a common type of variation in all the light curves.

This common type is indicated in the plates which accompany this paper.

Lovedale: 1902 *February* 1.

New Variable Stars found during the Measurements for the Astrographic Catalogue at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

In the course of the measurement of the plates for the photographic map of the heavens, under Mr. Hollis's direction, a careful note is made of large differences of diameter of images of the same star on the overlapping plates which are measured simultaneously in the duplex micrometer.

The following four stars were in this way found to be variable, and their diameter on other plates were measured; the variability of the third one was announced by Dr. Anderson in 1897. The other three appear to be new :—

		Approx. R.A. 1900. h m s	Approx. Dec. 1900. ° ′
A	... ...	18 5 9	65 56·9
B	... ...	18 6 54	66 8·9
C	U Draconis ...	19 9 42	67 7·1
D	... ...	5 49 22	74 30·8

A and B occur on the same plates; the following table gives their measured diameter (the unit being 0″·15) and that of neighbouring comparison stars whose magnitudes are taken from the *Bonn Durchmusterung* :—

No. of Plate.	Date.	Exp. m	A	B	66°1074 9·0	65°1238 9·5	66°1083 9·4	66°1079 9·5
411	1892 June 6	40	44	42	...	42	47	42
441	1892 „ 23	6	22	16	24	22	22	16
2117	1894 July 3	40	52	10	52	51	44	32
2688	1895 June 16	6	0	3	32	22	24	16
2697	1895 „ 17	6	0	0	...	25	22	13
2724	1895 „ 25	40	0	10	52	40	44	30
5088	1900 Sept. 13	6	16	0	22	22	22	20
5658	1901 June 6	40	21	3	41	33	38	26

A shows on these photographs changes of magnitude from nearly $9^{m}\cdot0$ to below the limit reached by chart plates of 40^{m} exposure or a nominal $14^{m}\cdot0$.

B shows almost as great a range, as it varies from $9^{m}\cdot5$ of the B. D. to nearly the limit of the chart plates.

The following table shows the variation of C (*U Draconis*) as shown by the measures of diameter :—

No of Plate.	Date.	Exp. m	C	67°1118 8·9	67°1120 9·2	67°1113 9·5
483	1892 Aug. 5	40	60	70	58	56
1380	1893 „ 19	6	18	46	38	39
1381	1893 „ 19	40	40	82	68	58
2136	1894 July 11	6	0	44	39	35
2288	1894 Oct. 16	6	4	33	32	19
3200	1896 July 12	40	3	80	64	52
5666	1901 „ 8	6	10	54	44	36

These measures show that C was only a little below $9^{m}\cdot0$ on 1892 August 5, and 1896 July 12 was only just shown on a chart plate of 40^{m} exposure.

The following table shows in a similar manner the variations in magnitude of D :—

No. of Plate.	Date.	Exp. m	D	74°275 8·1	74°267 8·2	74°272 9·3	74°266 9·5
3796	1897 Dec. 24	40	38	123	94	92	58
3869	1898 Feb. 23	40	20	103	84	78	62
3886	1898 Mar. 2	6	0	58	42	42	24
4302	1899 Feb. 25	6	40	54	42	34	22
5410	1901 Feb. 11	40	0	80	66	60	44

The above measures show that this star was between the 8th and 9th magnitudes on 1899 February 25, and on 1901 February 11 was below the limit of the chart plates.

The Magnitude of η Argûs 1900–1902. By R. T. A. Innes.

The two comparison stars used are given in the *Monthly Notices*, vol. lix. p. 570.

	Mag.	Colour.	Means. Date.	Mag.	Colour.
1900 Feb. 26	7·7	6			
Mar. 31	7·65	6	1900·3	7·68	6·3
June 8	7·7	7			

		Mag.	Colour.	Means. Date.	Mag.	Colour.
1901 Jan.	21	7·9	7			
Mar.	22	7·7	6·5	1901·3	7·78	6·8
June	8	7·75	7			
1902 Jan.	26	7·7	6·5			
Feb.	1	7·65	6·5	1902·1	7·72	6·5
	8	7·8	6·5			

Telescope, 7-in. Merz equatorial, as before.

Royal Observatory Cape of Good Hope:
1902 *February* 10.

Note on Professor Turner's Recent Paper on Photographic Surveying. By H. G. Fourcade.

(*Communicated by Sir David Gill.*)

In the *Monthly Notices* for December 1901 there appears a paper by Professor Turner, F.R.S., on "A Simple Method of Accurate Surveying with an Ordinary Camera."

The first method noticed by Professor Turner, in which a pair of views is taken at right angles to a base line, has, in its simple form, been used by me for some years, and, improved in respect of speed, convenience, and accuracy by the proposed adoption of a stereoscopic measuring instrument combining some of the features of stereoscopic range-finders and measuring machines for stellar photographs, it is described in a paper read before the South African Philosophical Society on October 2, 1901.

The second method in which the pair of views is taken in the direction of the base line was not considered by me, because it does not lend itself to stereoscopic measurement. It is due to Dr. G. Le Bon (*Les Levers Photographiques et la Photographie en Voyage*, 1889) and will be found described in J. A. Flemer's "Photographic Methods and Instruments" (App. No. 10, *U.S. Coast and Geodetic Survey Report for* 1897).

Forest Department, Cape Town:
1902 *February* 10.

The "Green Flash." By T. W. Backhouse.

I do not know whether there are different phenomena that go by this name, but having seen it several times when the Sun has been setting in the sea I can give a much simpler explanation of

it as seen by me than that given by W. H. Pickering (*Monthly Notices*, vol. lxii. p. 85).

When the Sun is seen through a telescope low down it is bordered at its lower edge, owing to refraction, with the red end of the spectrum, which looks on the whole red or orange-red, while at its upper edge it is bordered with the violet end, which is on the whole green, the extreme end of the spectrum being of course absorbed by the atmosphere. The edges necessarily boil much, but the last portion to set must naturally be a part of the green upper edge, and although I have not observed the green flash with the telescope it is obvious that it is this green edge that is last seen by the naked eye; the boiling of the edge will no doubt account for the appearance like Baily's beads described by one observer with instrumental means.

West Hendon House, Sunderland:
1902 *Jan.* 16.

Note on the "Green Flash" at Sunset.

By Major E. H. Hills, R.E.

The notes on this subject by Mr. W. H. Pickering (*Monthly Notices*, lxii. p. 85, 1901 November) and Mr. Backhouse above have suggested to me that it may be of some interest if I give a short account of my observations of this phenomenon and of the conclusions I have reached as to its cause. There is, I believe, no doubt that it is of spectral origin, the green being the visually predominant colour in a solar spectrum which attains the requisite degree of purity when the light source, namely the diminishing segment of the Sun's disc, becomes narrow.

If this explanation be the correct one, it will follow that, though the green may be by far the strongest colour, it will not be the last one visible, and a careful scrutiny, under suitable telescopic power, should reveal the more refrangible parts of the expiring spectrum, and thus, as the last portion of the Sun's rim finally disappears below the horizon, we should see the green fade away into blue and violet.

This I have actually observed at least twice, and I have no reason to doubt that it is visible at any time that the atmospheric conditions favour the appearance of a strong green flash.

In observing I used a Zeiss binocular, with a magnifying power of 12 diameters, and was careful to avoid eye fatigue by looking at the Sun only when a very small portion remained above the horizon. Under these circumstances the narrowing segment, which is, of course, of an orange-red colour, is first seen to turn green at its two extremities; the green colour then appears to run inwards until the whole area of the segment appears coloured.

This is the moment when it is visible to the naked eye as the green flash, and after a duration of about a second the green fades away and is momentarily replaced by a blue or blue-violet. The latter is immeasurably more feeble than the green, and its visibility only lasts a small fraction of a second, but it is quite unmistakable, and its appearance seems to prove beyond any question the spectral character of the colours seen.

Further evidence, all tending in the same direction, is found in the fact that the green flash is better seen when the eye has not been previously tired by looking at the red disc of the Sun, whereas if it were a complementary, or fatigue, colour a certain amount of previous gazing is absolutely necessary, and also in the well-authenticated appearance of the same phenomenon at sunrise (see *Nature*, vol. lx. p. 511, 1899 August 31, where Lord Kelvin gives an account of its being seen by him at sunrise in Switzerland), when any retinal fatigue is out of the question. The conditions favourable for its appearance would seem to be :—

1. An atmosphere free from haze and dust; otherwise the light of short wave length will be dispersed and lost, and the green thus made faint or altogether obliterated.
2. A distant horizon, whether land or sea is immaterial ; but, as a dusty atmosphere is practically impossible over the sea, it is on the latter that the phenomenon is generally noticed.

   The distance of the horizon line is an important point. If the air were perfectly free from dust and haze it would be immaterial, within large limits, where the horizon is situated, but practically it is necessary to get as long a column of unilluminated air in front of the observer as possible, and therefore the more distant the horizon the more likely is the flash to make its appearance. I have, for instance, repeatedly tried to see it with a land horizon at about two miles distance, and have, under these circumstances, always failed.
3. A still atmosphere and one homogeneous as to its temperature, or in other words conditions identical with those necessary for good "seeing" in a telescope ; otherwise the irregular dispersion of the air will mask the purity of the colours.

Condition No. 2 leads us up to the fact that the best conceivable circumstances for seeing it would be when the Sun sets behind the horizon at the maximum possible distance—namely, the Moon—provided that the observer is so situated that the Sun is at a very low altitude at the time of eclipse.

Observers of eclipses have naturally always placed themselves so as to get the Sun at as high an altitude as possible, so that the appearance of the green flash at the moment of second or third contact in the case of an annular or total solar eclipse has not yet been recorded.